DIRECT CURRENT LIMITING FUSES

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DIRECT CURRENT LIMITING FUSES

PETER G. BEIERL
JOHN R. BAYLIS

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Monterey, California

DIRECT CURRENT LIMITING FUSES

by

Lieutenant Peter G. Beierl, U.S. Navy

B.S., U.S. Naval Academy, 1944

and

Lieutenant (junior grade) John R. Baylis, U.S. Navy

B.S., U.S. Naval Academy, 1945

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

NAVAL ENGINEER

from the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

1951

Cambridge, Massachusetts
18 May, 1951

Professor J. S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the degree of Naval Engineer, a thesis entitled, "Direct Current Limiting Fuses" is herewith submitted.

Respectfully,

Peter G. Beierl
Lieutenant, U.S. Navy

John R. Baylis
Lieutenant (junior grade), U.S. Navy

ACKNOWLEDGMENT

The authors are indebted to Professor E. W. Boehne for his invaluable guidance in the supervision of this thesis.

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SUMMARY

DIRECT CURRENT LIMITING FUSES

by

Lieutenant Peter G. Beierl, U.S. Navy

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Lieutenant (junior grade) John R. Baylis, U.S. Navy.

Submitted for the degree of Naval Engineer in the Department of Naval Architecture and Marine Engineering on 18 May 1951.

When we are interested in limiting the magnitude of fault current in direct current systems, the use of current limiting fuses has basic advantages. It is often impossible to provide any current limiting effect in circuit breakers, and current limiting will often save weight and space by decreasing the interrupting capacity required for the circuit.

The design and development of limiters has been hampered by the lack of an adequate basis for evaluation of development tests. The results of this thesis provide ~~and~~ method of comparison of fuse tests and will, when coupled with test data, provide a method of determining the most severe conditions a given fuse will encounter. The results of this thesis can be used to analyze any fuse whose voltage waveform can be approximated by a linearly rising voltage.

The method of graphical calculation illustrated here can be used to extend the analysis to any simple fuse voltage waveform. This thesis shows that for the voltage waveform assumed, and for one possible mode of variation of fuse arc voltage, there is one fault condition which produces

the most severe conditions at the fuse. This is the condition at which production tests should be conducted.

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INTRODUCTION

The Naval Problem

The problem of direct current limiters and fuses has been thrust upon the naval engineer by the development of high submerged speed submarines which depend on storage batteries for propulsive power when submerged. It is not the propulsion circuit which poses the problem, but the circuit as installed in the submarine, for in the submarine we must provide circuit protection within the weight and space allotted. This is a problem in naval architecture and we will not pursue this point further than to state that weight and space are extremely critical in submarine design.

In naval applications, circuit protection is best provided by circuit breakers. Breakers provide the greatest continuity of service, ease of restoration of service, and control of interrupting characteristics. However, breakers are inherently handicapped by the minimum time lag of their mechanical operation. In many circuits the time constant is small enough to permit the current to rise to maximum short circuit value before the breaker will initiate interruption. Therefore, in these applications, breakers in their present state of development cannot limit the magnitude of fault current. A breaker with the capacity to interrupt the maximum fault current on a modern submarine occupies too much space, and we must look for alternative schemes. We cannot leave the circuit unprotected, for if the batteries are allowed to discharge uncontrolled, there is a great possibility that one or more cells will reverse polarity.

Cell reversal is usually accompanied by a dangerous release of energy and gas. In addition, the maximum available currents involved are prohibitively destructive. This has led to the consideration of current limiting fuses for use in submarines. Limiters are fuses which are designed only to limit the magnitude of short circuit currents. They do not protect against overload. It is expected that a limiter can provide protection with the least cost in weight and space.

Determination of Criteria

Generally, fuses fail by rupture of the case, restriking, or by dielectric breakdown either at the fuse or in some other protion of the circuit. Baxter (10) gives some illustrations which bear this out. Whenever the casing of the fuse fails during interruption we can assume that the pressure and temperature inside the casing became too great. This may be due to an inadequacy in the casing, but, except for such structural defects, it is caused by the release of energy in the arc beyond the energy absorbing capacity of the filler. If this excess energy sufficiently heats the insulating materials of the fuse, they may become conducting and cause reignition of the arc, or restrike. Excessive arc energy, then, is one of the causes of fuse failure and the amount of energy liberated in the fuse arc can be used as a measure of the difficulty of interruption.

To rapidly interrupt the current in an inductive circuit we must produce a large voltage to oppose the current. Stated differently, a rapid interruption of the current produces large voltages. If the voltages are too great, insulation may break down, either at the fuse or in some other

part of the circuit. Since we must limit the voltages produced in the circuit to prevent damage to insulation, the maximum voltage produced across the fuse can be used to judge the performance of the fuse. Based on the above, we have chosen as criteria for evaluating current limiting fuse performance, the fuse arc energy and the maximum fuse arc voltage. We recognize that restrike may be caused by voltage transients initiated at current interruption, but we have not extended the analysis to a study of these transients except as inferred in the evaluation of the maximum voltage across the fuse.

The Physics of the Arc

The physics of the electric arc has been studied by some of the world's foremost physicists. For a review of the work done in this field the reader is referred to an excellent article by Spraragen and Lengyel entitled "The Physics of the Arc" (1). The work done in this field has defined the behavior of the arc under static conditions (2), (3), (4) but does not define the conditions in the transient arc sufficiently for engineering application.

Interrupters

A great deal of progress has been made in the interruption of the A.C. arc, where attention has naturally been focussed on the region of current zero. (5), (6). The methods of A.C. circuit interruption may be classified as either low arc voltage or high arc voltage interruptions. The methods used in analysis of low arc voltage interruptions do not apply to the study of direct current interrupters since this principle depends on part of the circuit. Since we must limit the voltages produced in the circuit to prevent damage to insulation, the maximum voltage produced across the fuse can be used to judge the performance of the fuse. Based on the above, we have chosen as criteria for evaluating current limiting fuse performance, the let-thru energy and the maximum fuse arc voltage. We recognize that restrike may be caused by voltage transients initiated at current interruption but we have not extended the analysis to a study of these transients except as inherent in the evaluation of the maximum voltage across the fuse.

The Physics of the Arc

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Interruption

A great deal of research has been done on the interruption of the A.C. arc where attention has naturally been focused on the region of current zero. [illegible]. The processes of A.C. circuit interruption may be classified as either low arc voltage or high arc voltage interruption. The methods used in analysis of low arc voltage interruption [illegible] the study of direct current interruption [illegible] the current zeros produced by the applied alternating voltage. We can, however, adapt some of the techniques used in the analysis of the high arc voltage A.C. interrupters. In the analysis of interrupters, Boehne has considered the arcs as voltage sources; (7), (8) and this method can easily be applied to the study of direct current interrupters. Boehne and Jang (9) have, in this way, formulated performance criteria for direct current interrupters. They have shown that the most advantageous voltage waveform is rectangular and they have concentrated on this form.

Fuses

Work published in the study of fuses has not been as extensive as that on circuit breakers. Baxter has recently written a book (10) which is chiefly concerned with the physical construction of fuses and the effects of the variables of construction on fuse performance. Boehne and Shuck have established performance criteria for A.C. current limiting power fuses (11), (12). They have shown that there is a "most severe" condition of these fuses and they have advocated testing at this condition.

So far as we can determine, there has been no published investigation of current limiting fuses for direct current service. The fuse designer has little to guide him, and fuse design has proceded empirically, guided only by experience. Progress has been hampered by the lack of an adequate basis for evaluation of development tests. That is, a fuse was either successful or unsuccessful; there has been no easy means to determine the relative difficulty of interruption.

Purpose of Investigation

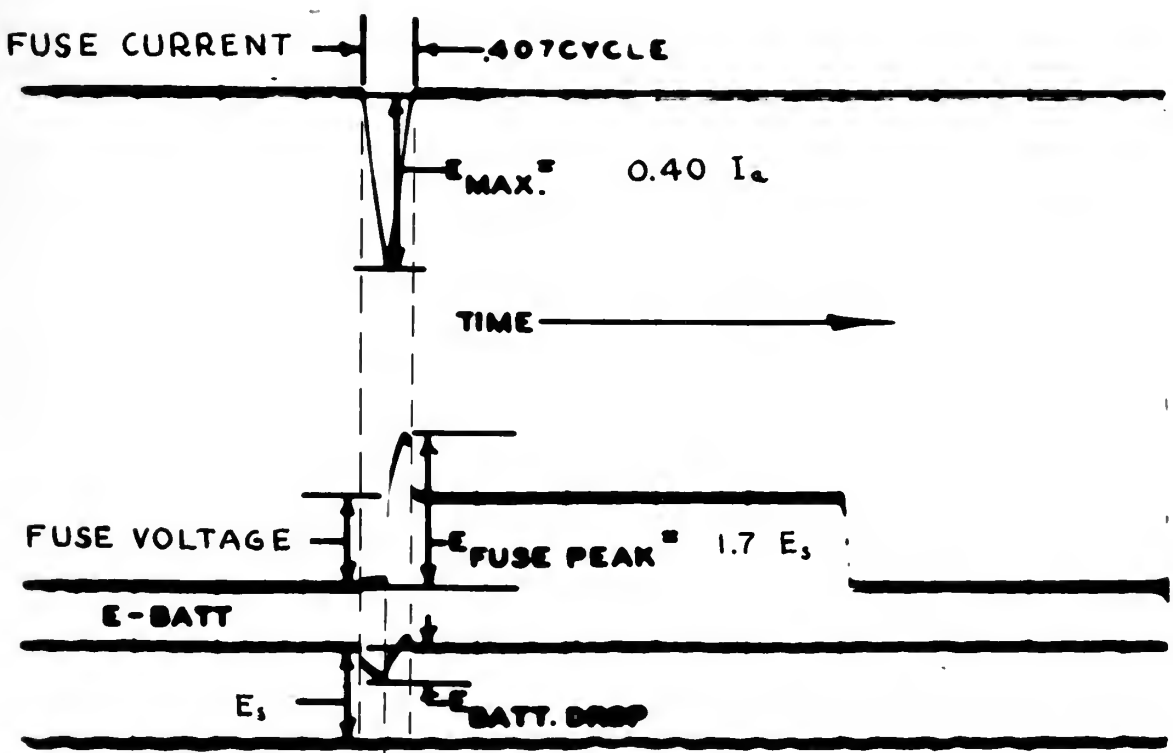
We intend to provide a means of evaluating fuse tests and a method of determining the most severe conditions a fuse will encounter. We hope that this information, together with the fuse designers experience, will indicate what steps must be taken to improve fuse performance. We don't intend to show how to build a fuse to meet a set of specifications, but only to provide a means of comparing fuses, and to provide a means of predicting the effect of altered conditions on a given fuse.

To accomplish this, we will assume that the voltage produced by a fuse is of some simple form which approximates voltages found in fuse tests. We will then use the type of super-position analysis employed by E. W. Boehne in his technical papers. (6), (7), (8), (9), (12).

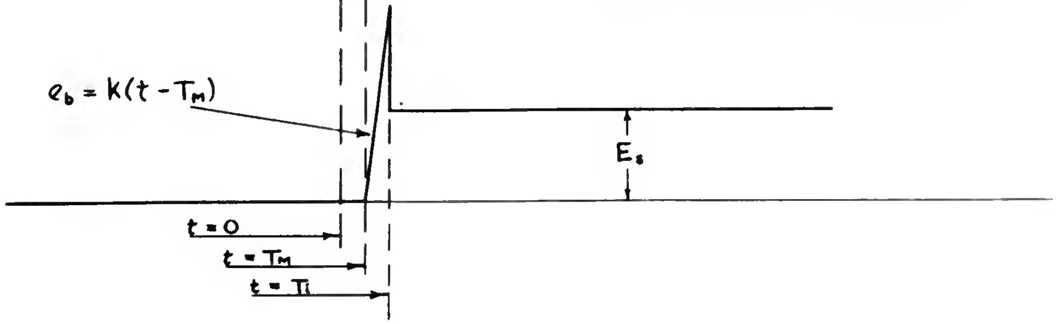
FIGURE I

APPROXIMATION OF FUSE ARC VOLTAGE

OSCILLOGRAM OF CURRENT LIMITING
FUSE INTERRUPTION



THE VOLTAGE APPROXIMATION



PROCEDURE

General

To attack the problem we have assumed linear lumped parameter circuits in which the fusable element is approximated by a pure time-variant voltage source. This makes the problem linear and greatly amplifies the mathematics. The character of the fuse voltage was studied by observing oscillograms of fuse interruptions; and a linearly rising voltage was selected as a useful approximation of the voltage waveform produced by many interrupters (see figure I). It is true that a trapezoidal voltage is a closer approximation in many instances. However, the added variable of maximum voltage greatly complicates the problem of analysis, and in most cases the advantage of better voltage approximation is decreased because the net fuse current is close to zero in the region of voltage maximum.

We have normalized the analysis to make it immediately useful, regardless of the magnitude of the particular parameters involved. Since the circuit was made linear by describing the fuse arc as a pure voltage source, the principle of superposition may be applied.

It was assumed for all the calculations that the circuit external to the limiter was a series combination of a voltage source which produced a step voltage at the time of fault, a lumped circuit resistance, and a lumped circuit inductance. Capacitance was assumed to be negligibly small.

General

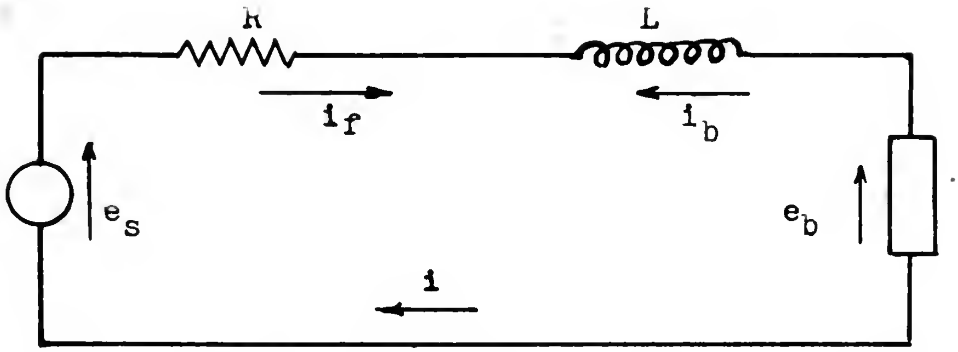
To attack the problem we have assumed [illegible] circuits in which the fusable element is [illegible] variant voltage source. This makes the circuit [illegible] amplifies the mathematics. The character [illegible] by observing oscillograms of fuse interruption [illegible] voltage was selected as a useful approximation [illegible] produced by many interrupters (see figure [illegible]) [illegible] voltage is a closer approximation in many instances [illegible] variable of maximum voltage [illegible] and in most cases the advantage of [illegible] voltage [illegible] creased because the net fuse current is [illegible] voltage maximum.

We have normalized the [illegible] regardless of the magnitude of the [illegible] the circuit was [illegible] voltage source, the principle of [illegible]

It was assumed for all [illegible] the limiter was a series [illegible] etc. voltage at the [illegible] circuit inductance [illegible]

FIGURE II

CIRCUIT FOR CASE I, SINGLE FUSE ELEMENT LIMITER

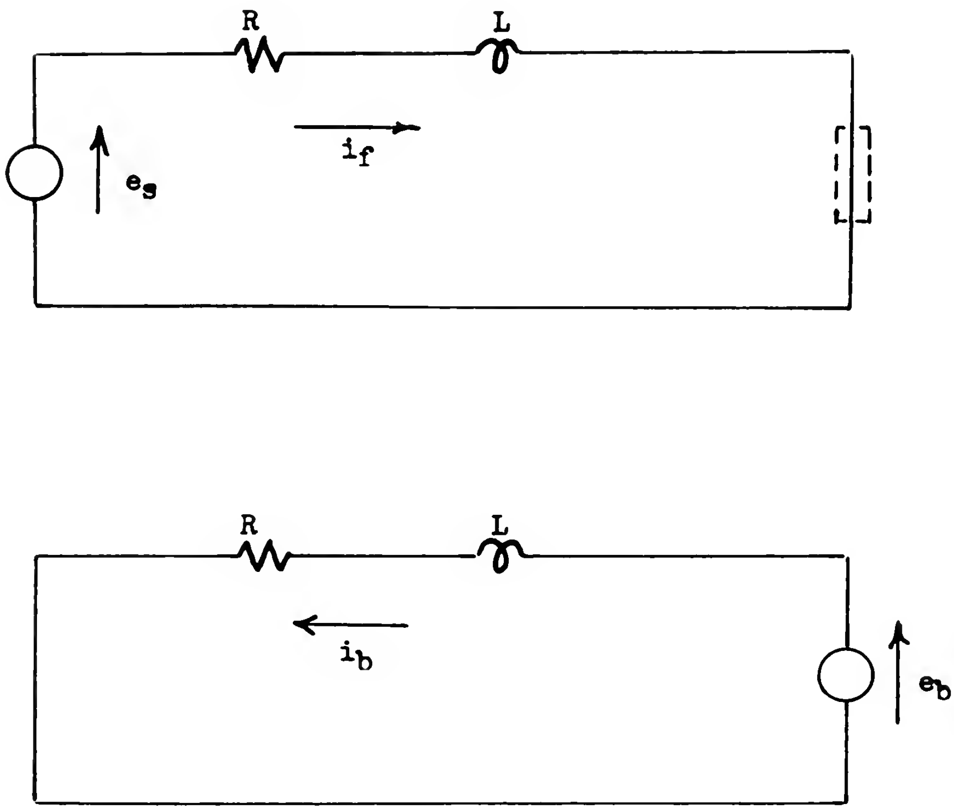


DEFINITIONS

Total Circuit Resistance,	R		
Total Circuit Inductance,	L		
Circuit Time Constant,	τ	=	L/R
Available Current,	I_a	=	E_s/R
Forward Current,	i_f	=	$E_s/R(1-e^{-t/\tau})$
Melting Time	T_m		
Fuse Voltage	e_b	=	$k(t-T_m)$
Back Current	i_b	=	$k/R(t-T_m - \tau e^{-(t-T_m)/\tau} - \tau)$
Total Current	i	=	$i_f - i_b$
Fuse Arc Energy	W		
Melting Energy	M		
Rate of Rise of Fuse Voltage	k		

FIGURE II A

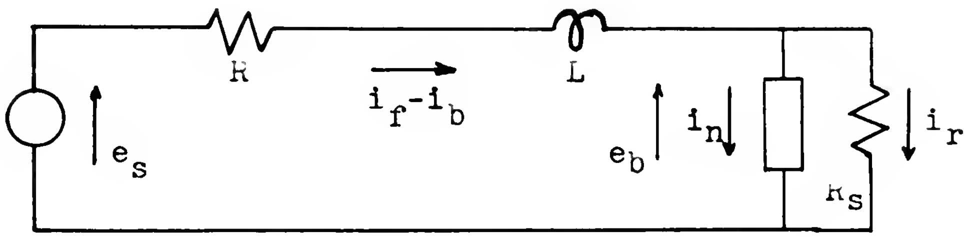
METHOD OF SUPERPOSITION FOR CASE I



Net fuse current: $i = i_f - i_b$

FIGURE III

CIRCUIT FOR CASE II, SHUNTED FUSE LIMITER

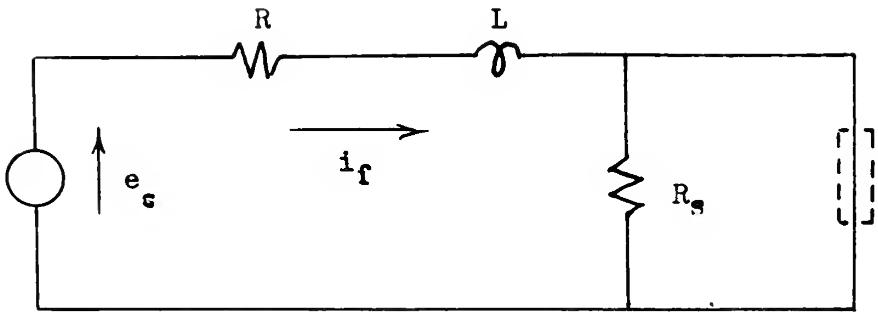


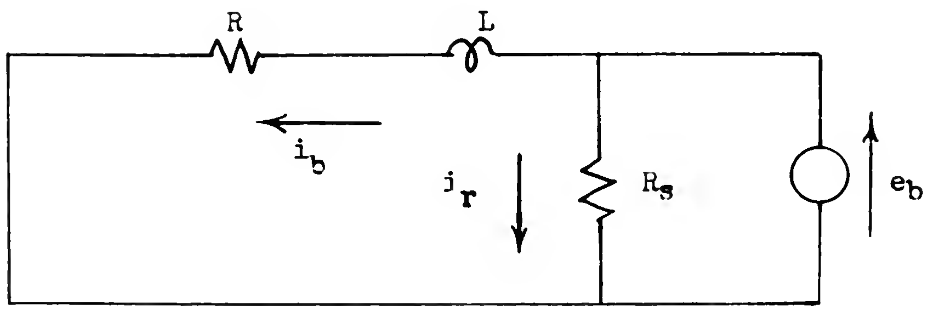
DEFINITIONS

Total Circuit Resistance	R
Total Circuit Inductance	L
Circuit Time Constant	$\tau = L/R$
Available Current	$I_a = E_s/R$
Forward Current	$i_f = E_s/R(1-e^{-t/\tau})$
Melting Time	T_m
Fuse Voltage	$e_b = k(t-T_m)$
Back Current	$i_b = k/R(t-T_m - \tau e^{-(t-T_m)/\tau} - \tau)$
Shunt Resistance	R_s
Shunt Current	$i_r = e_b/R_s$
Net Fuse Current	$i_n = i_f - i_b - i_r$
Rate of Rise of Fuse Voltage	k
Fuse Arc Energy	W
Melting Energy	M

FIGURE III A

METHOD OF SUPERPOSITION FOR CASE II

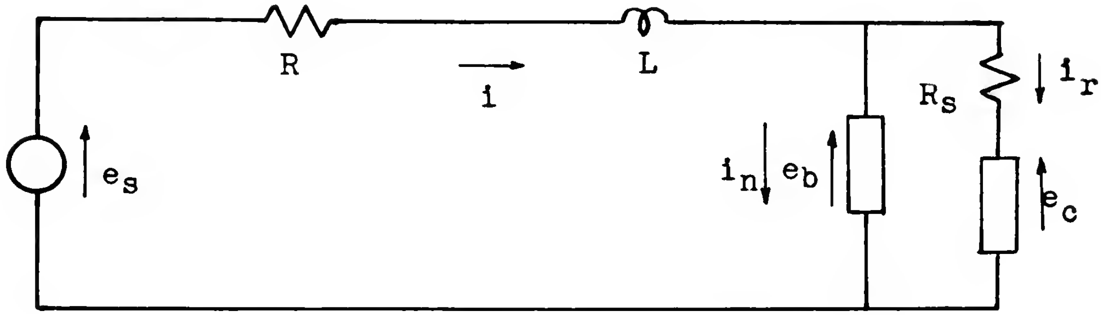




Net fuse current: $i_n = i_f - i_b - i_r$

FIGURE IV

CIRCUIT FOR CASE III, PARALLEL THREE-ELEMENT LIMITER



DEFINITIONS

Total Circuit Resistance	R
Total Circuit Inductance	L
Circuit Time Constant	$\tau = L/R$
Available current	$I_a = E_s/R$
Forward Current	$i_f = E_s/R(1-e^{-t/\tau})$
Melting Time	Tm
First Fuse Voltage	$e_b = k_1(t-T_m)$
Shunt Resistance	R_s
Shunt Current	i_r
Net Current, Shunted Fuse	i_n
Fuse Arc Energy	W
Melting Energy	M
Rate of Rise of Fuse Voltage	k
Second Fuse Voltage	$e_c = k_2(t-T_{m2})$
Back Current, Fuse 2	i_c

FIGURE IV A

METHOD OF SUPERPOSITION FOR CASE III

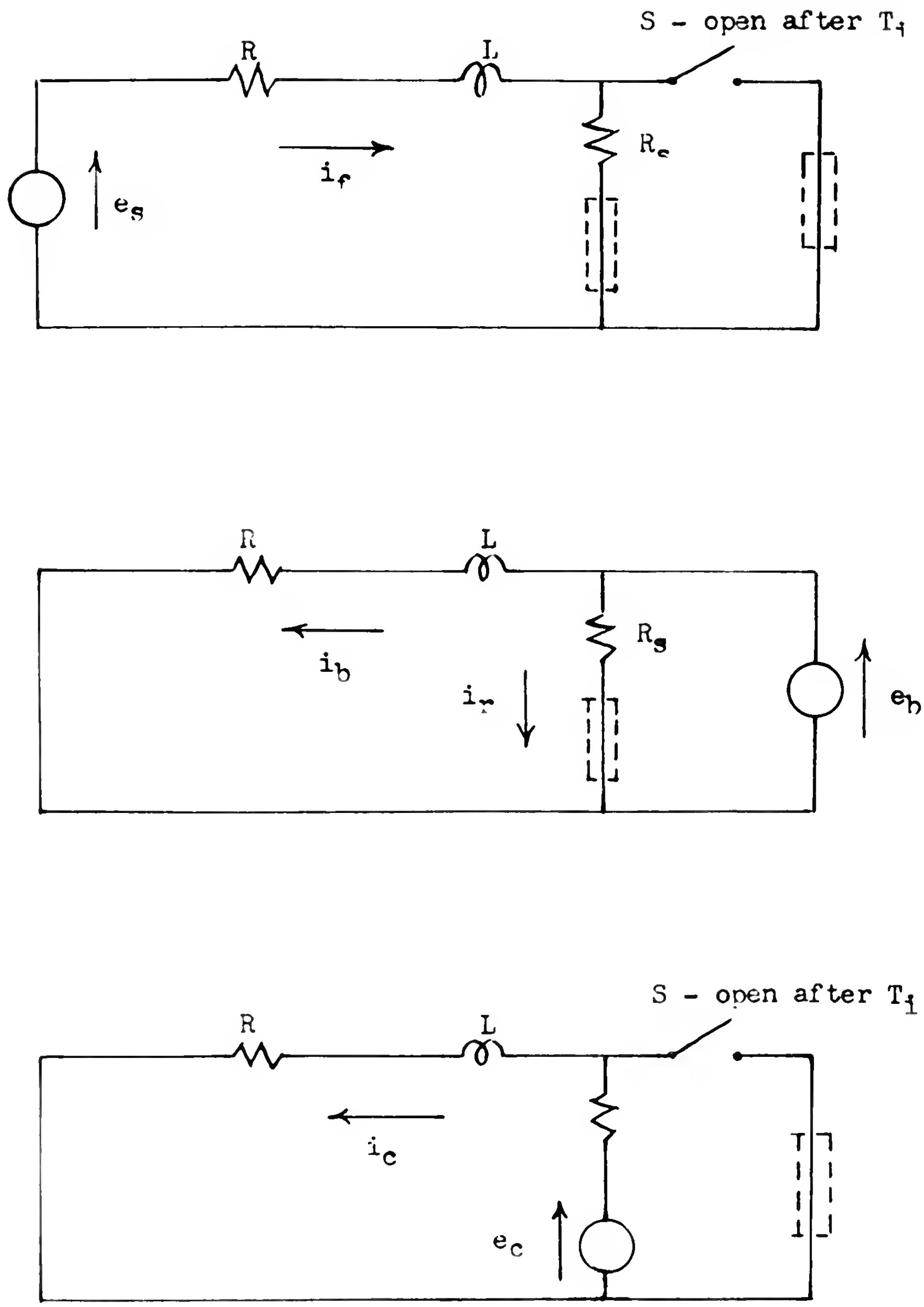
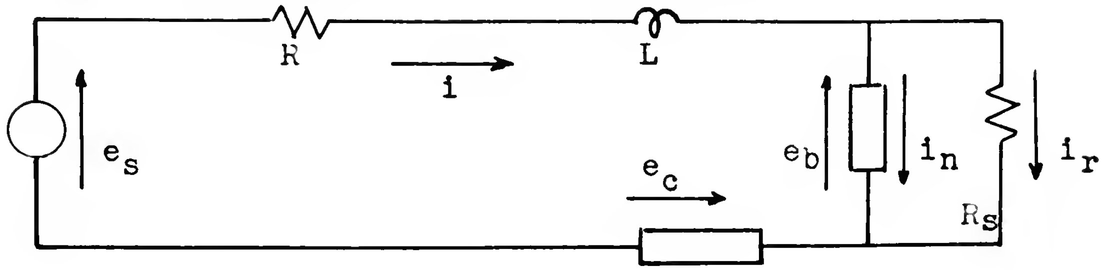


FIGURE V

CIRCUIT FOR CASE IV., SERIES THREE-ELEMENT LIMITER

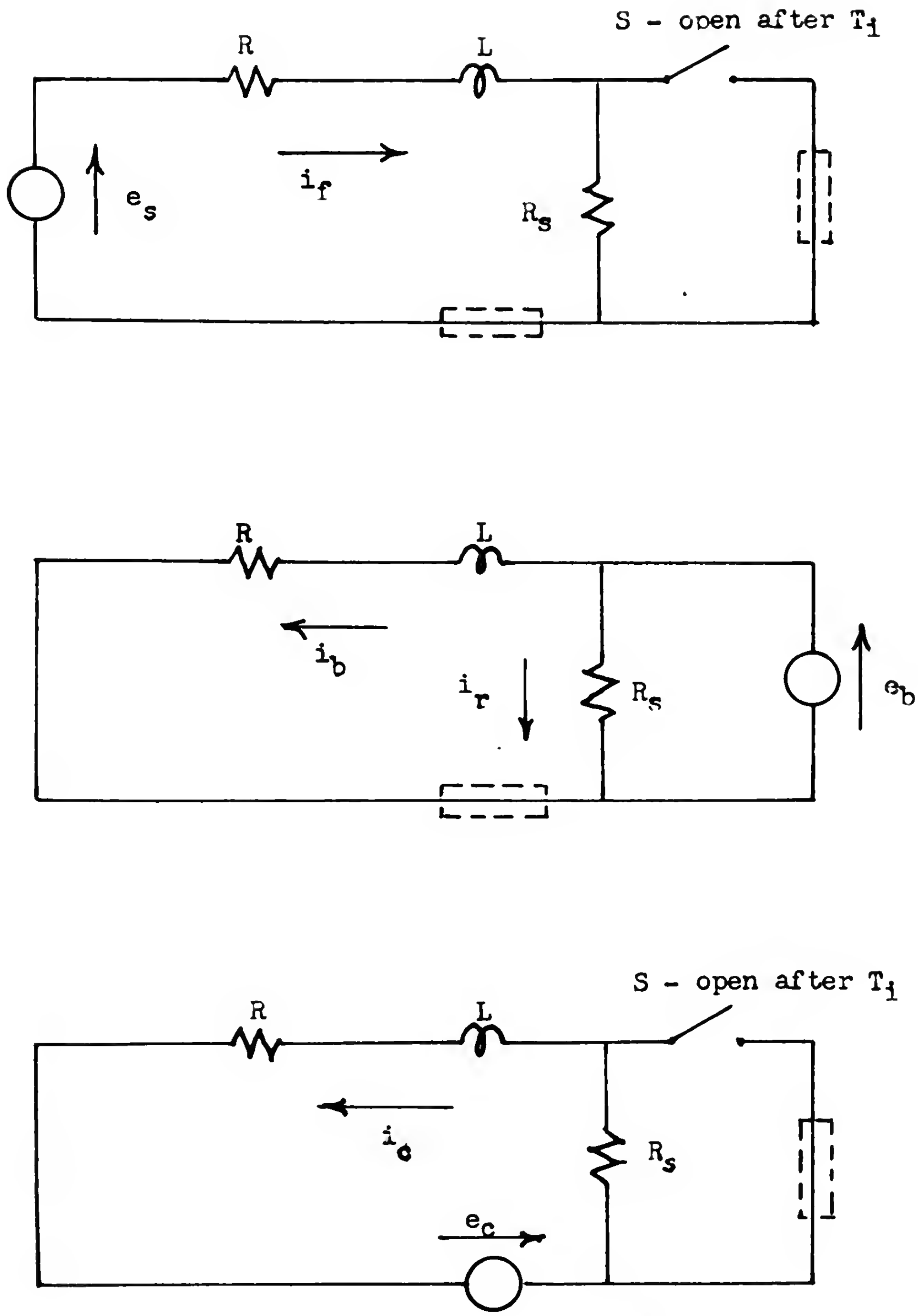


DEFINITIONS

Total Circuit Resistance	R
Total Circuit Inductance	L
Circuit Time Constant	$\tau = L/R$
Available Current	$I_a = E_s/R$
Forward Current	$i_f = E_s/R(1-e^{-t/\tau})$
Melting Time	T_m
First Fuse Voltage	$e_b = k_1(t-T_m)$
Shunt Resistance	Rs
Shunt Current	i_r
Net Current, Shunted fuse	i_n
Fuse Arc Energy	W
Melting Energy	M
Rate of Rise of Fuse Voltage	k
Second Fuse Voltage	$e_c = k_2(t-T_{m2})$
Back Current, Fuse 2	i_c

FIGURE V A

METHOD OF SUPERPOSITION FOR CASE IV



<u>Limiter Arrangements</u>

Four limiter arrangements were considered. Case I was taken as a single fuse element limiter, approximated by a pure voltage source which produces a linearly rising voltage beginning at the instant of melting (see figure II). For Case II the limiter was considered to be the fusable element just described, shunted by a resistance, R_s (see figure III). In Case III the limiter was a resistance shunted fusable element in series with a second fusable element in series with a second fusable element (see figure IV). Case IV was likewise a three-element limiter, but with the second fuse in series with the resistor shunt, in the leg parallel to the first fuse (see figure V).

Upon examination of the equations expressing the net fuse currents and fuse energies (see Appendix A), it was apparent that the problem of setting the limits for the required integrations would make the mathematical computation of usefully large numbers of samples extremely laborious. In order to accomplish our objective in the time available, we have resorted to graphical calculation and integration by planimeter. While this introduced small errors from the mechanical integration and the translation of graphical values, it prevented gross errors by providing a continuous plot of the voltages, currents, and power. The superposition principle was used wherever possible in solving for the net fuse currents because this allowed the advantageous use of graphical overlays in the computation process. The general method of superposition for Cases I through IV is indicated by figures IIA, IIIA, IVA, and VA, respectively.

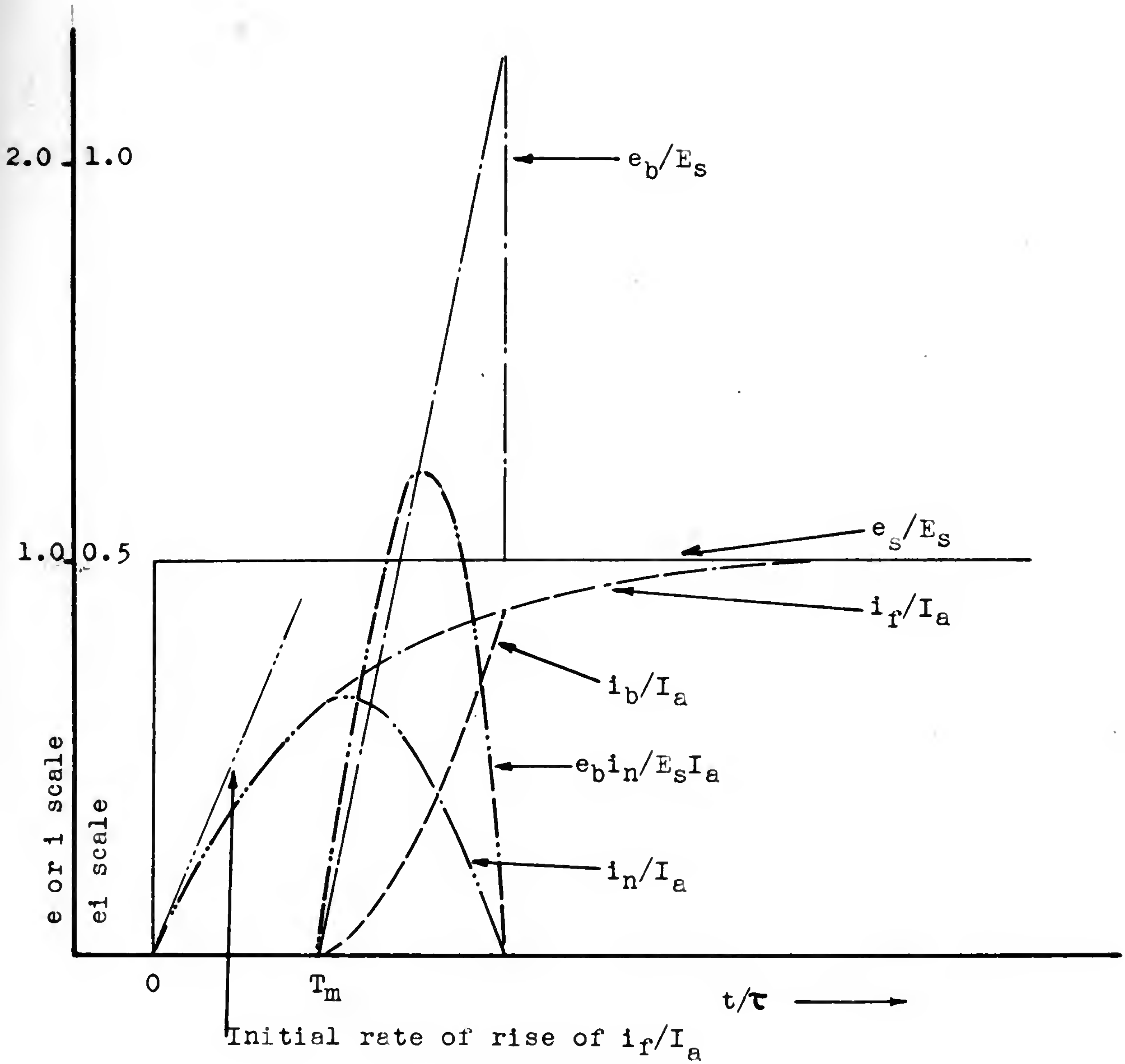
Limiter Arrangements

Four limiter arrangements were considered. Case I was taken as a single fuse element limiter, approximated by a pure voltage source which produces a linearly rising voltage beginning at the instant of melting (see figure II). For Case II the limiter was considered to be the fusible element just described, shunted by a resistance R_s (see figure III). In Case III the limiter was a resistance shunted fusible element in series with a second fusible element (see figure IV). Case IV was likewise a three-element limiter, but with the second fuse in series with the resistor shunt in the [illegible] (see figure V).

Upon examination of the equations [illegible], relating the [illegible] fuse current and fuse energies (see [illegible]) it was [illegible] setting the limits for the required integrations would [illegible] computation of usefully large numbers [illegible] In order to accomplish our objective in the time available [illegible] to graphical calculation and integration [illegible] duced small errors from the mechanical [illegible] of graphical values [illegible] plot of the voltages, currents, [illegible] used wherever possible in solving [illegible] allowed [illegible] process. [illegible] indicated by figures II [illegible]

FIGURE VI

METHOD OF CALCULATION, CASE I



DEFINITIONS

E_s	Source Voltage	i_b	Back Current
I_a	Available Current	i_n	Net Fuse Current
i_f	Forward Current	τ	Circuit Time Constant
e_b	Fuse Voltage	T_m	Melting Time

Case I

The details of the graphical computation for Case I are illustrated by figure VI. The method for the other three arrangements is identical in principle. The net fuse current (i_n) is obtained by subtracting the back current (i_b) due to fuse voltage, (e_b) from the forward current (i_f) due to source voltage (e_s). The arc energy ( ) is determined by mechanically integrating the product of fuse voltage and net current.

Case II

For Case II, the net fuse current is obtained by subtracting the shunt resistor current (i_r) from the resultant current of Case I. Maximum circuit current is taken from the results of Case I.

Case III

In the desirable mode of operation xx for Case III, Fuse 2 melts at some time after the completion of the fuse interruption. In the normal mode of operation, the voltage and energy for Fuse 1 can be obtained from Case II, while the voltage and energy for fuses 2 can be obtained from Case I. The critical value of melting energy for Fuse 2 may be computed as a simple function of i_r during the interval between melting and interruption for Fuse 1. Appendix C contains a formula expressing this critical melting energy as a function of Case II maximum fuse voltage. The chief interest in the second fuse centers on the peak voltage developed, because this voltage is impressed across Fuse 1 and directly affects the possibility of restrike in Fuse 1. Since no current flows in Fuse 2 before Fuse 1 melts, there is small probability of Fuse 2 melting before interrup-

tion is complete. Therefore, the range of investigation was not extended to include the effect of early melting of Fuse 2.

Case IV

In the desired mode of operation for Case IV, as in Case III, Fuse 2 melts at some time after interruption 1 is completed. For this normal operation, as before, the energy and maximum voltage for Fuse 2 are taken from Case I. The critical values of melting energy for Fuse 2 are computed from the resultant circuit current (i) of Case II, from the time of fault initiation to the end of interruption 1. The values of Fuse 2 melting energies for T_2 equal to T_1 plus 2 (in the early melting computation) approximate the critical values closely enough to show trends, hence the exact critical values are not separately calculated. The chief interest for the second fuse is in the arc energy, since the maximum voltage of Fuse 2 does not affect Fuse 1 directly.

Because the total circuit current always flows through Fuse 2, it is quite probable that under some conditions the second fuse will melt before interruption 1 is complete. Should Fuse 2 melt before this critical time, it is apparent that interruption 1 ends earlier and that the arc energy of Fuse 1 is decreased, since the net current through Fuse 1 is reduced by the back current due to the voltage of Fuse 2. For Fuse 2 melting energies less than the critical value the arc energy of Fuse 2 is determined by graphic computation. In brief, the Fuse 2 back current is subtracted from the total current through Fuse 1 and resistor shunt of Case II. The discontinuity in the resistance parameter occurring when Fuse 1 is tion is complete. Therefore, the range of [illegible]

to include the effect of [illegible]

Case IV

In the desired range of [illegible]

melts at some [illegible]

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energies [illegible]

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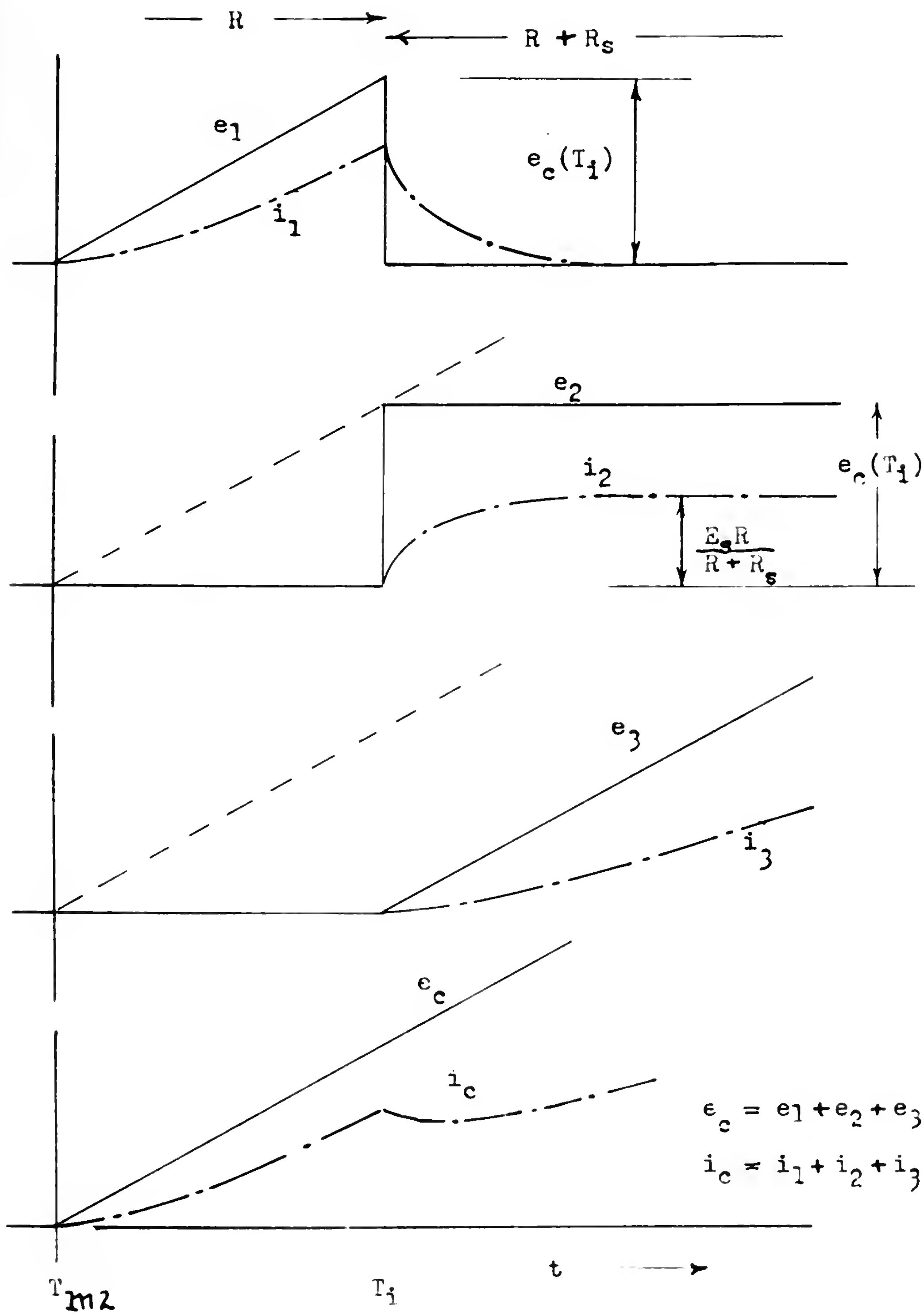
FIGURE VII

CALCULATION OF BACK CURRENT FOR FUSE 2 VOLTAGE, CASE IV



removed from the circuit modifies the total current from Case II slightly, and complicates the Fuse 2 back current. This back current was calculated, as shown in Figure VII, by superposition of two current components: 1) that for Fuse 2 voltage with initial circuit resistance, terminating when net current in Fuse 1 reaches zero (T_i): 2) that due to Fuse 2 voltage after T_i, with resistance equal to the sum of initial circuit and shunt resistances.

Normalization

Quantities entering into the computation were normalized as follows:

Currents - divided by available fault current (I_a)

Voltages - divided by the source voltage magnitude (E_s)

Time - divided by the circuit time constant (τ)

Arc energies ($W = \int ei\, dt$ for fuse arc) - divided by product of source voltage, available current, and circuit time constant ($E_s I_a \tau$).

Melting energies ($M = \int i^2\, dt$ for fusable element) - divided by the product of circuit time constant and the square of available current ($I_a^2 \tau$).

Slope of fuse voltage characteristic (k) - dividied by the ratio of source voltage to circuit time constant (E_s/τ).

Since the results will be in the normalized form, it is important to note here that in computing the effect of a change in any parameter the change in the base quantities used in normalization must be considered. While this attaches some additional labor to interpretation of the results, it is necessary in order to extend their application over an unlimited range.

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Normalization

Quantities entering into the computation were normalized as follows:

Currents - divided by available fault current (I_a).

Voltages - divided by the source voltage magnitude (E_a).

Time - divided by the circuit time constant (T).

Arc energies ($\int ei\, dt$ for fuse arc) - divided by product of source voltage, available current, and circuit time constant ($E_a I_a T$).

Melting energies ($\int i^2 dt$ for fusible element) - divided by the product of circuit time constant and the square of available current ($I_a^2 T$).

Slope of fuse voltage characteristic (K) - divided by the ratio of source voltage to circuit time constant (E_a/T).

Since the results will be in the normalized form, it is important to note here that in computing the effect of a change in any parameter, the changes in the base quantities used in normalization must be considered. Also this attaches some additional labor to interpretation of the results. It is necessary in order to assure their application over an unlimited range.

RESULTS

Case I

The results of computation for Case I are contained in figures VIII through XII, as listed below:

Figure VIII: Contours of constant arc energy as a function of melting energy and fuse voltage slope; for melting energies from .02 to 2.5, slopes from .25 to 5.0.

Figure IX: Contours of constant maximum fuse voltage as a function of melting energy (from .02 to 2.5) and slope of fuse voltage (from .25 to 5.0).

Figure X: Maximum current as a function of melting energy (from .02 to 2.5) for constant values of fuse voltage slope (from .25 to 5.0).

Figure XI: Arc energy as a function of melting energy (from .02 to 2.5), for constant fuse voltage slope.

Figure XII: Maximum voltage as a function of melting energy for constant values of fuse voltage slope.

Case II

The computations for Case II, the shunted fuse, were completed for shunt resistance (R_s) for 0.5 to 4.0 times the circuit resistance (R), fuse voltage slopes form 0.5 to 2.0, and melting energy from .02 to 2.5. The results are plotted in figures XIII through XVII, as listed below:

Figure XIII: Contours of constant arc energy as a function of shunt resistance (from 1/2 to 4) and melting energy (from

.02 to 2.5), for particular values of fuse voltage slope (1/4 to 2).

Figure XIV: Interpolation curves for variation of arc energy with fuse voltage slope at constant shunt resistance and melting energy, for use with Figure XIII.

Figure XV: Maximum fuse arc voltage as a function of shunt resistance for constant melting energy and rate of rise of arc voltage.

Case IV

The results of the computations for early melting are contained in figures XVIII through XX. For particular values of Fuse 1 voltage slope and shunt resistance, Fuse 2 energy is shown as a function of Fuse 2 slope and Fuse 2 melting energy.

Table I shows a normalized tabulation of approximate critical Fuse 2 melting energy as a function of shunt resistance (R_s/R), Fuse 1 melting energy ($M/I_a^2\tau$), and Fuse 1 voltage slope ($k\tau/E_s$).

.02 to 2.5), for particular values of fuse voltage slope (1/4 to 2).

Figure XIV: Interpolation curves for variation of arc energy with fuse voltage slope of constant shunt resistance and melting energy, for use with Figure XIII.

Figure XV: Maximum fuse arc voltage as a function of shunt resistance for constant melting energy and rate of rise of arc voltage.

Case IV

The results of the computations for early melting are contained in figures XVIII through XX. For particular values of Fuse 1 voltage slope and shunt resistance, Fuse 2 energy is shown as a function of Fuse 2 slope and Fuse 2 melting energy.

Table I shows a normalized tabulation of approximate critical Fuse 2 melting energy as a function of shunt resistance (R_s/R), Fuse 1 melting energy ($M/\lambda_s^2 C$), and Fuse 1 voltage slope ($k\sqrt{LC}/E_s$).

FIGURE VIII

CONTOURS OF ARC ENERGY AS A FUNCTION OF MELTING ENERGY AND SLOPE OF FUSE VOLTAGE

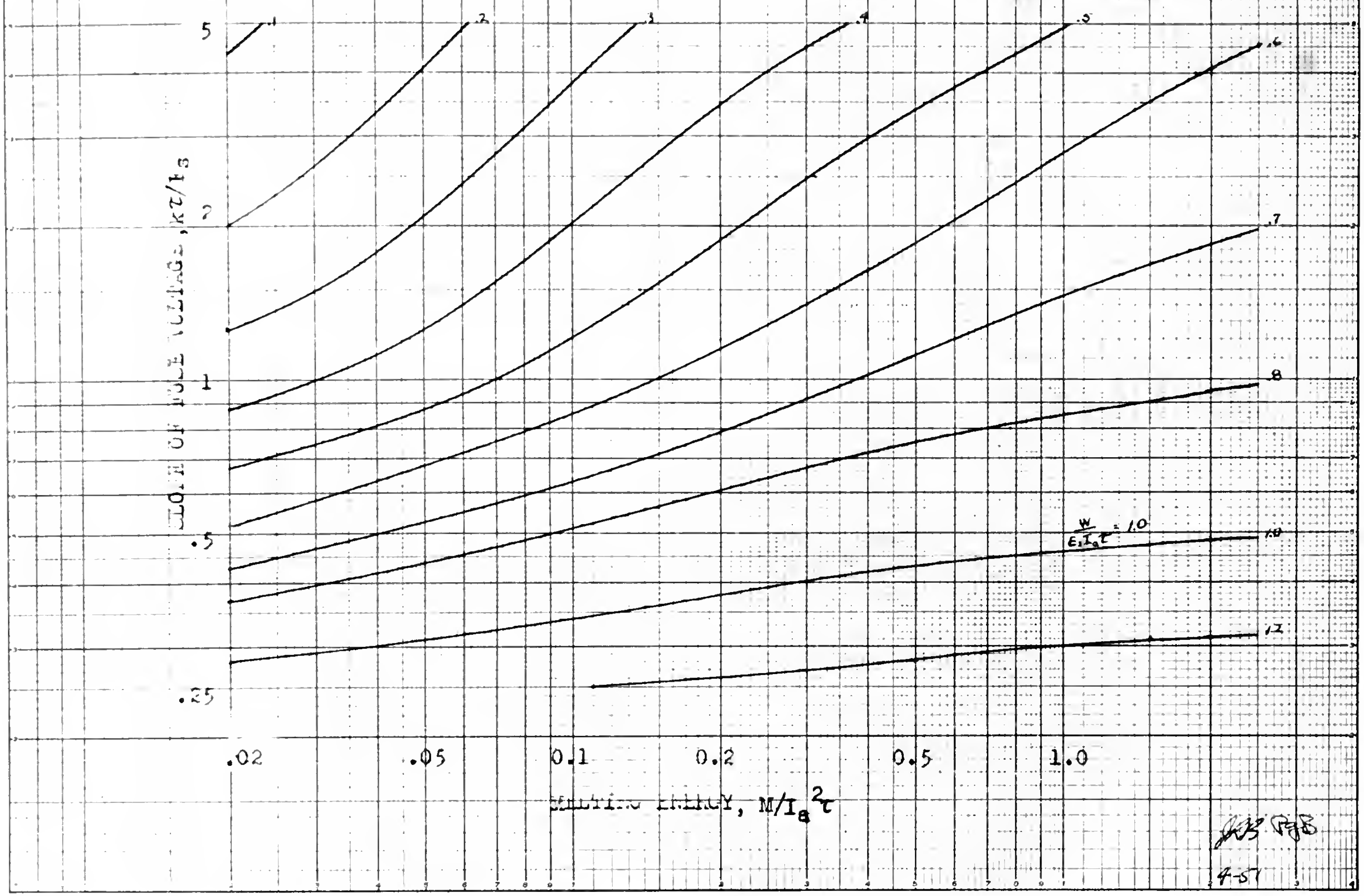

FIGURE IX

CONTOURS OF MAXIMUM ARC VOLTAGE AS A FUNCTION OF MELTING ENERGY AND SLOPE OF FUSE VOLTAGE

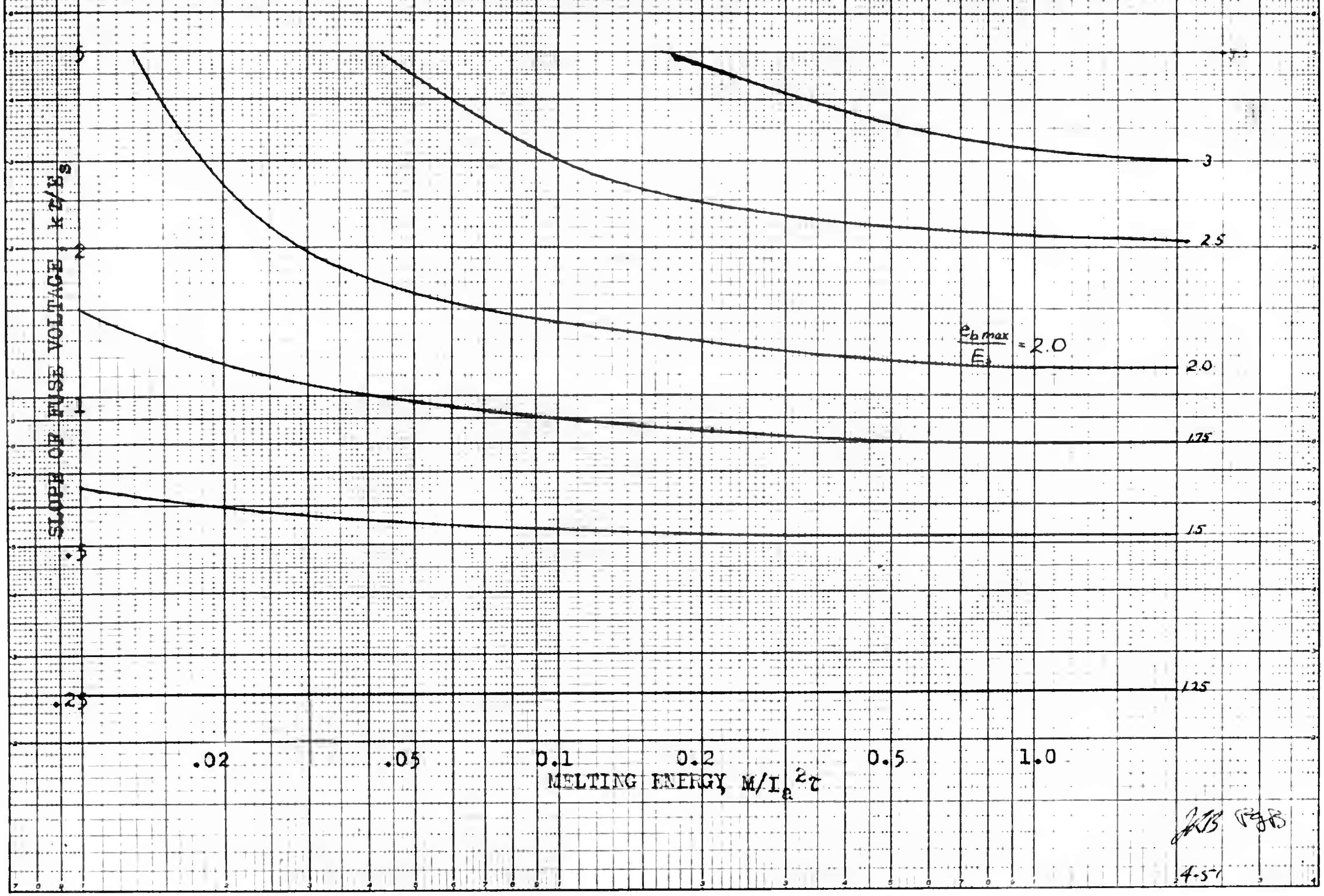

FIGURE X

MAXIMUM CURRENT AS A FUNCTION OF MELTING ENERGY FOR CONSTANT SLOPE OF FUSE VOLTAGE

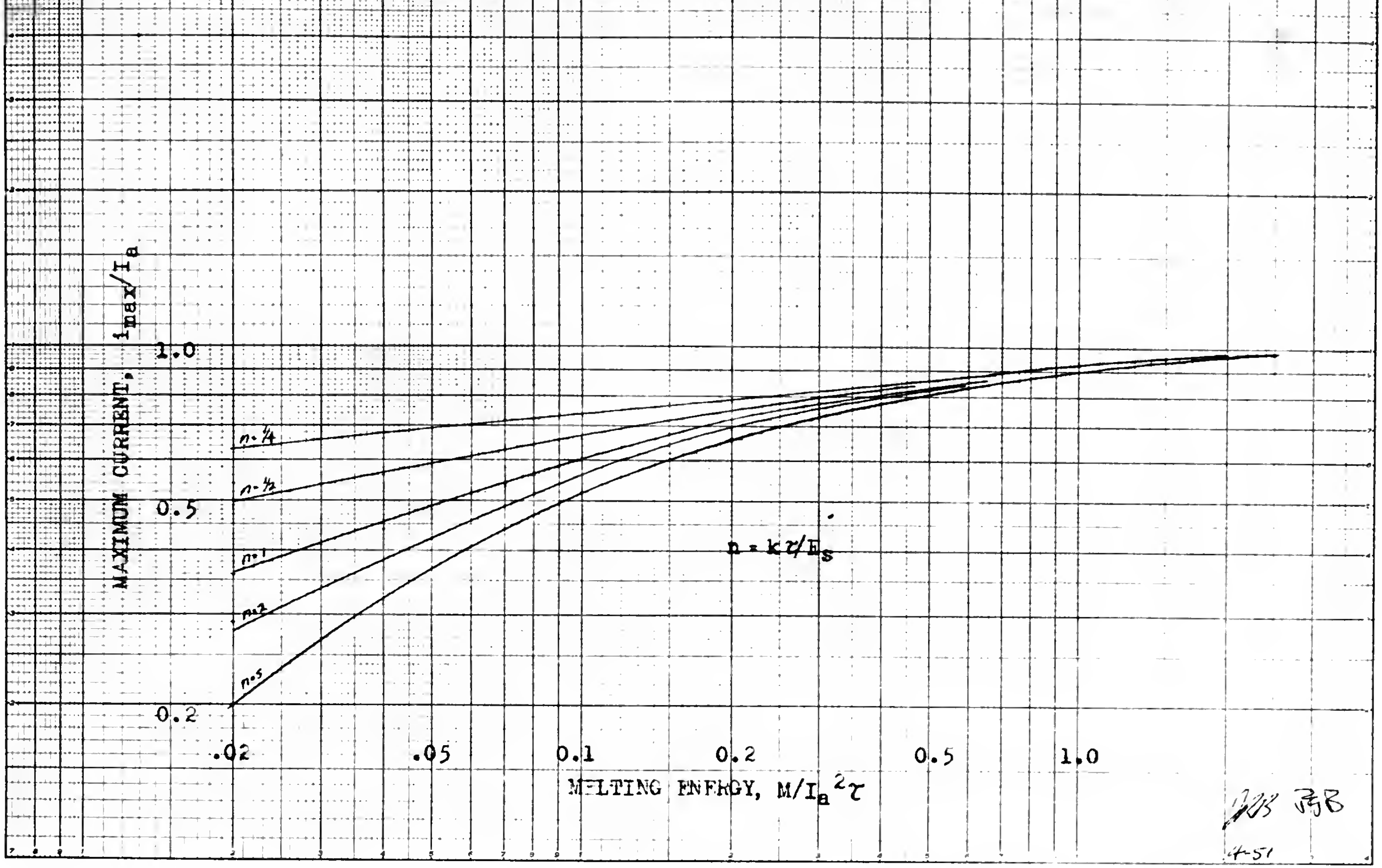

FIGURE XI

ARC ENERGY AS A FUNCTION OF MELTING ENERGY FOR CONSTANT SLOPE OF FUSE VOLTAGE

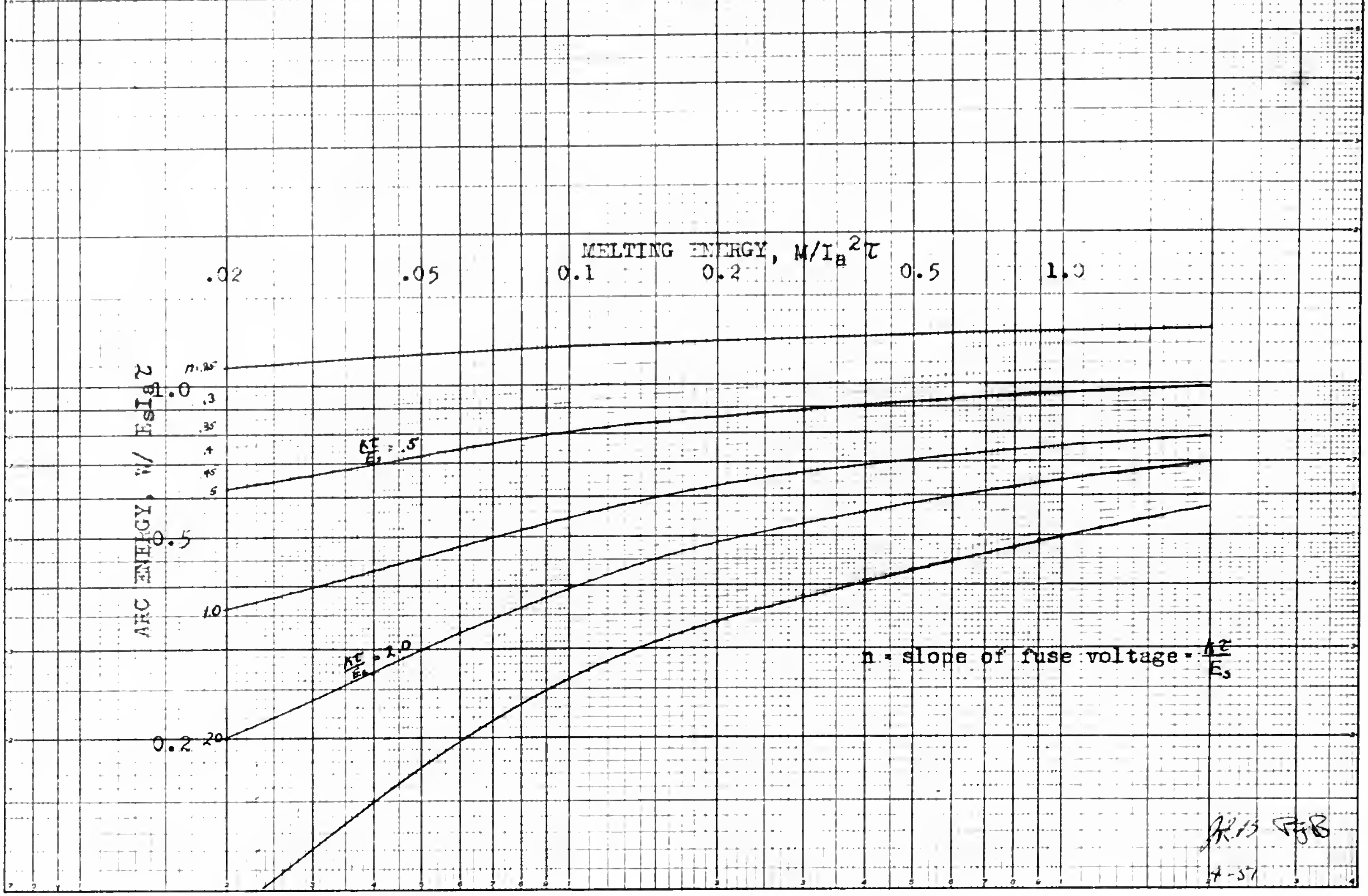

FIGURE XII

MAXIMUM FUSE VOLTAGE AS A FUNCTION OF MELTING ENERGY FOR CONSTANT SLOPE OF FUSE VOLTAGE

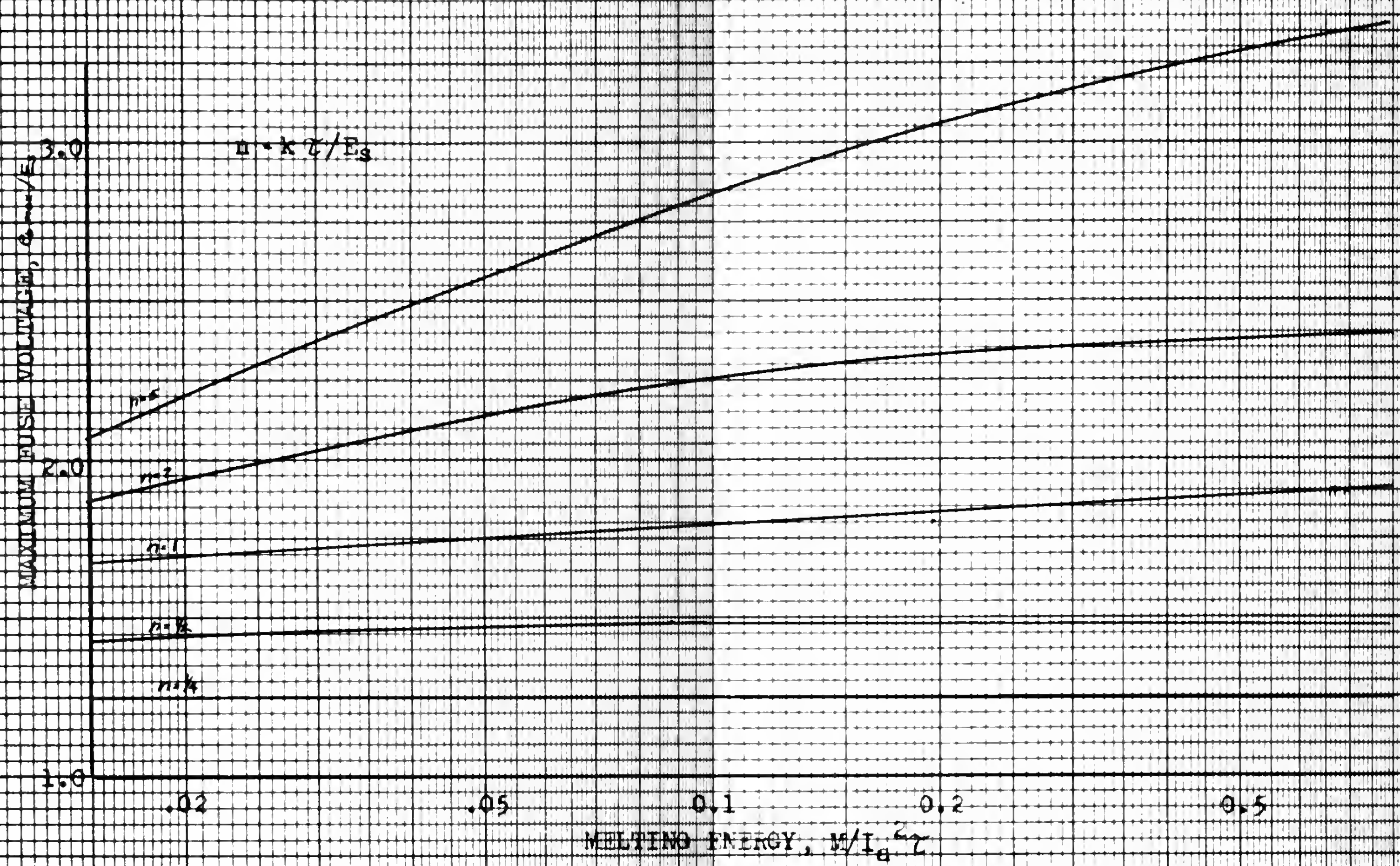

FIGURE XIII A

CONTOURS OF CONSTANT ARC ENERGY AS A FUNCTION OF SHUNT RESISTANCE AND MELTING ENERGY

SLOPE OF ARC VOLTAGE, $k\tau/E_s$, EQUALS TWO

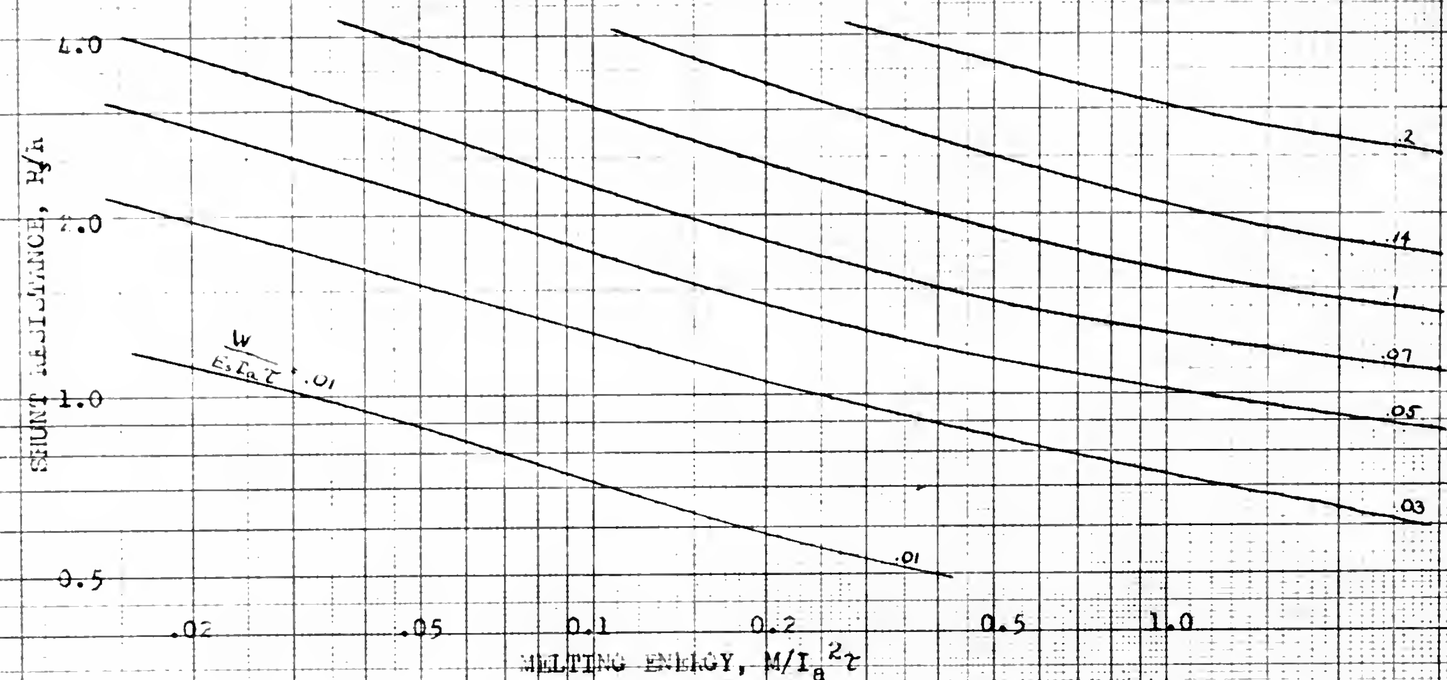

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FIGURE XIII B

CONTOURS OF CONSTANT ARC ENERGY AS A FUNCTION OF SHUNT RESISTANCE AND MELTING ENERGY

SLOPE OF ARC VOLTAGE, $k\tau/E_s$, EQUALS ONE

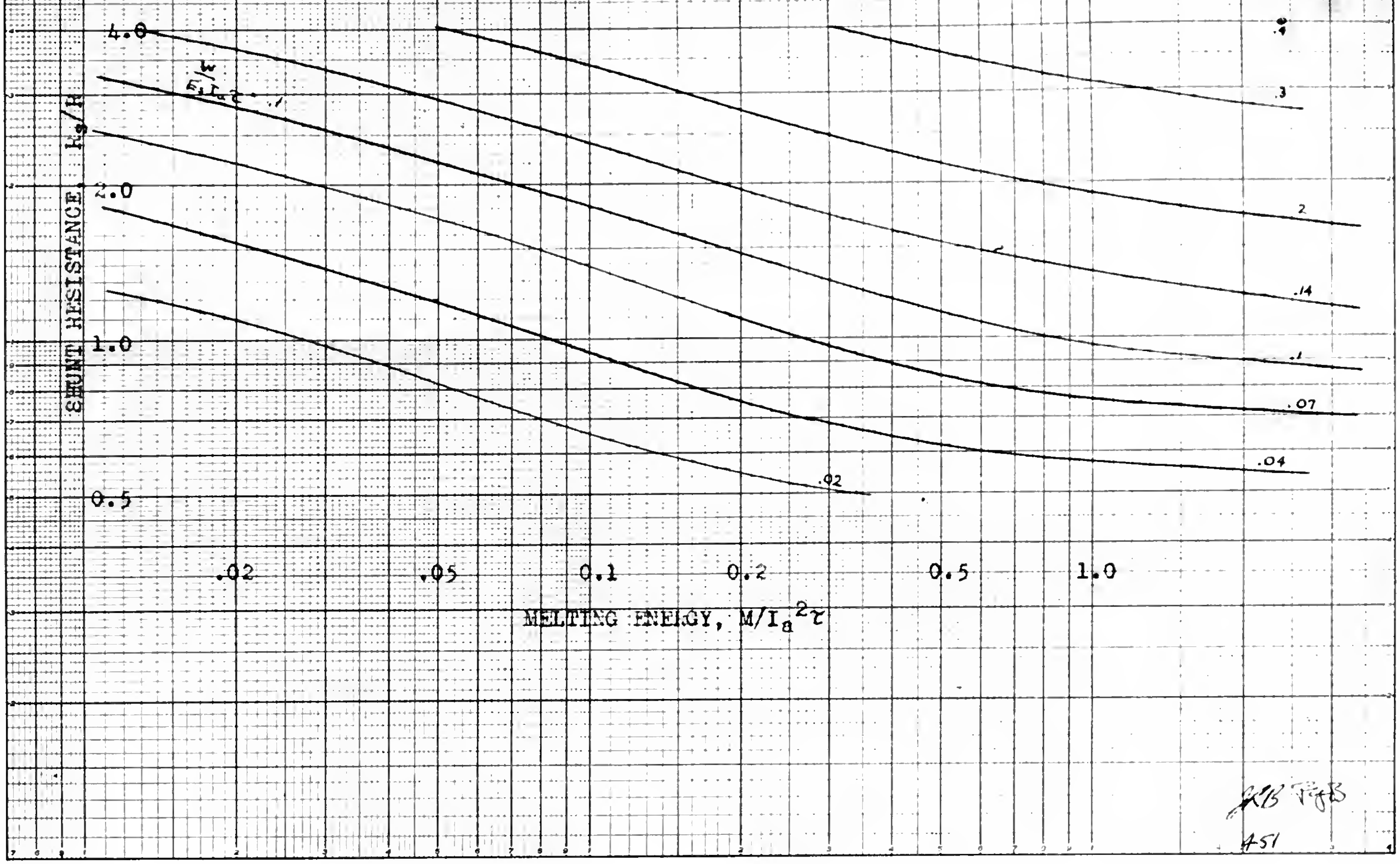


FIGURE XIII C

CONTOURS OF CONSTANT ARC ENERGY AS A FUNCTION OF SHUNT RESISTANCE AND MELTING ENERGY

SLOPE OF ARC VOLTAGE, $k\tau/E_s$, EQUALS ONE HALF

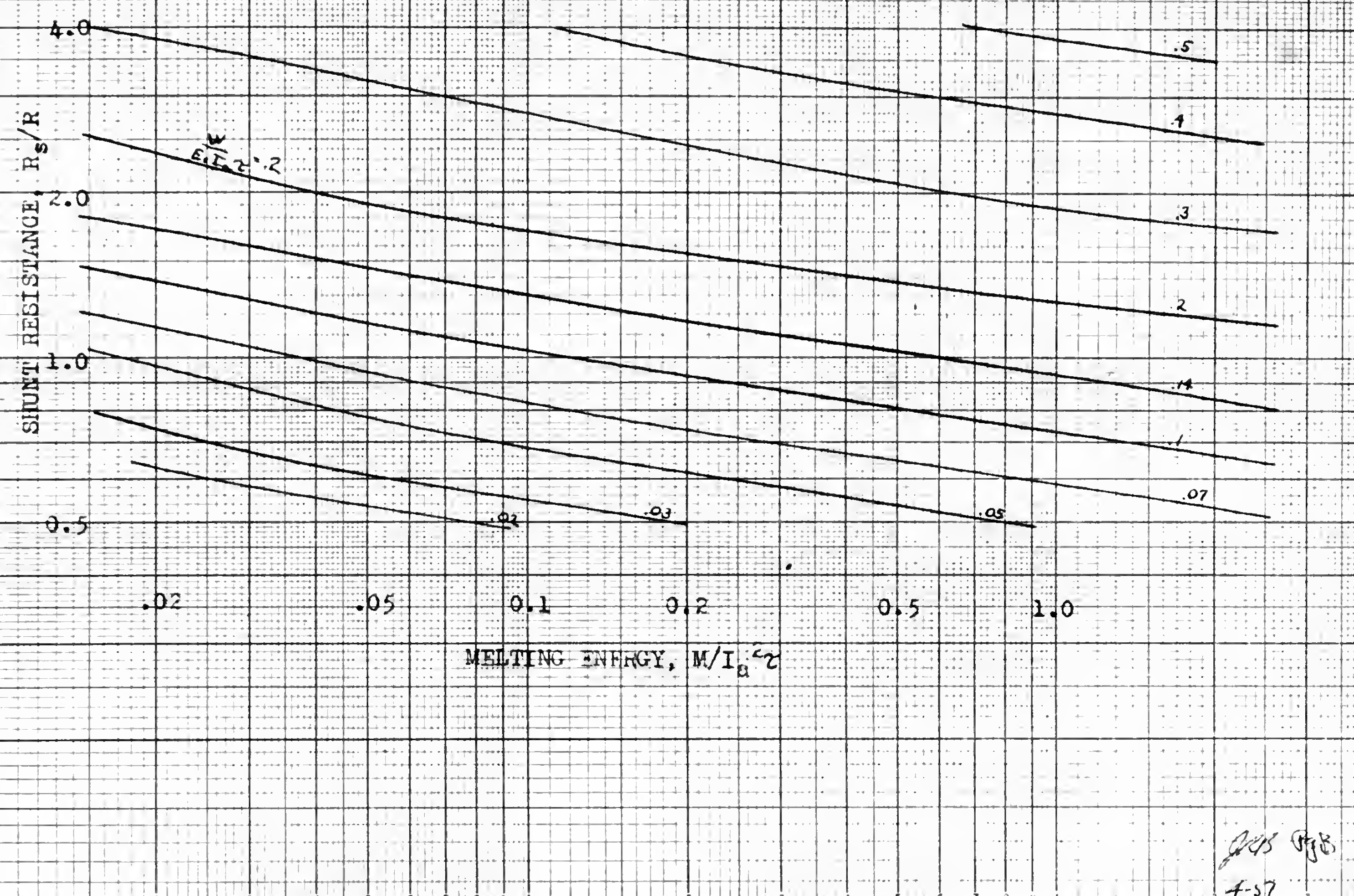
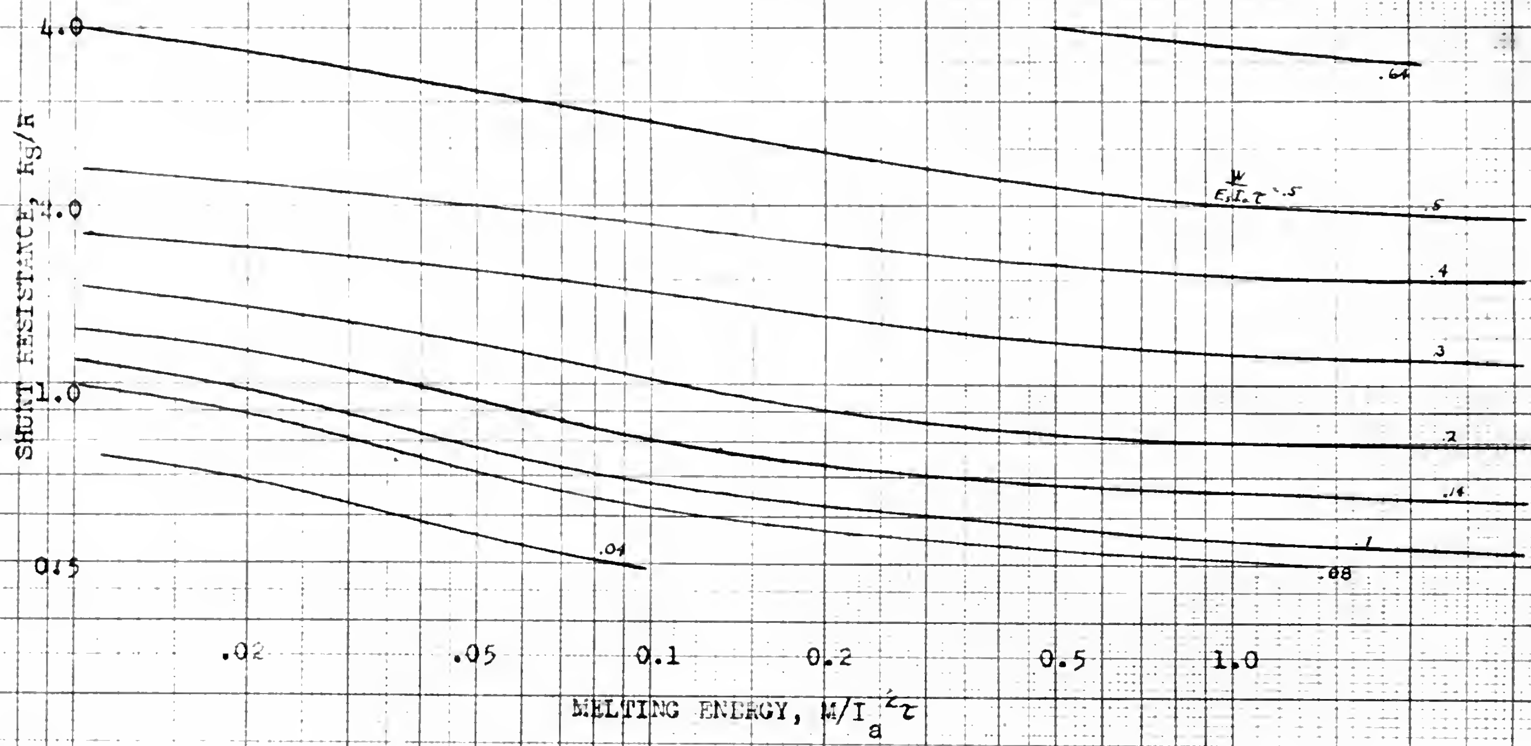


FIGURE XIII D

CONTOURS OF CONSTANT ARC ENERGY AS A FUNCTION OF SHUNT RESISTANCE AND MELTING ENERGY

SLOPE OF ARC VOLTAGE, $k\tau/E_s$, EQUALS ONE QUARTER



RB RB

4-57

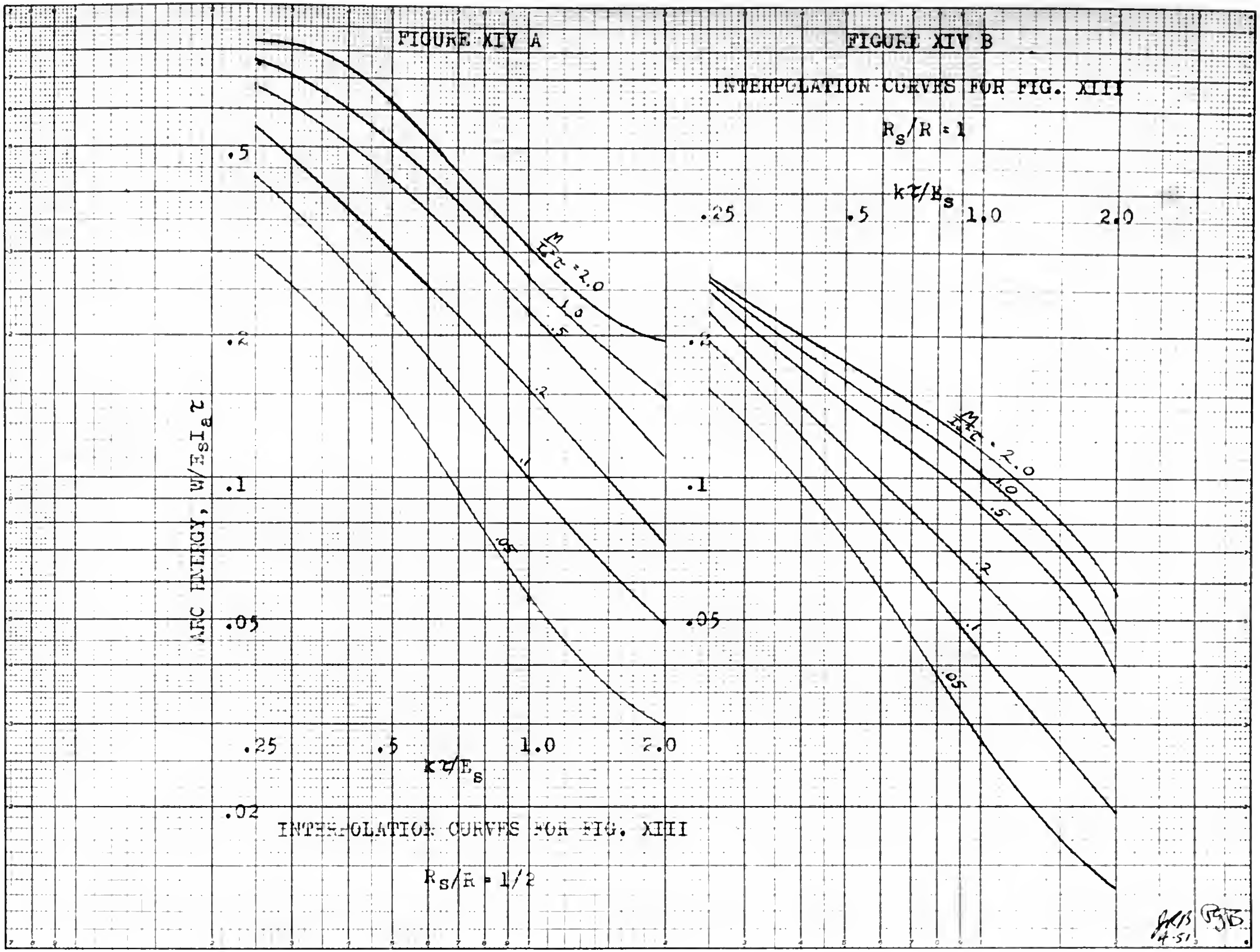


FIGURE XIV A
FIGURE XIV B
INTERPOLATION CURVES FOR FIG. XIII
$R_s/R = 1$
$k\tau/E_s$
.25
.5
1.0
2.0
$\frac{M}{I_s \tau} = 2.0$
1.0
.5
.2
.1
.05
ARC ENERGY, $W/E_s I_s \tau$
.5
.2
.1
.05
.02
INTERPOLATION CURVES FOR FIG. XIII
$R_s/R = 1/2$

FIGURE XIV C

INTERPOLATION CURVES FOR FIG. XIII

$R_s/R = 2$

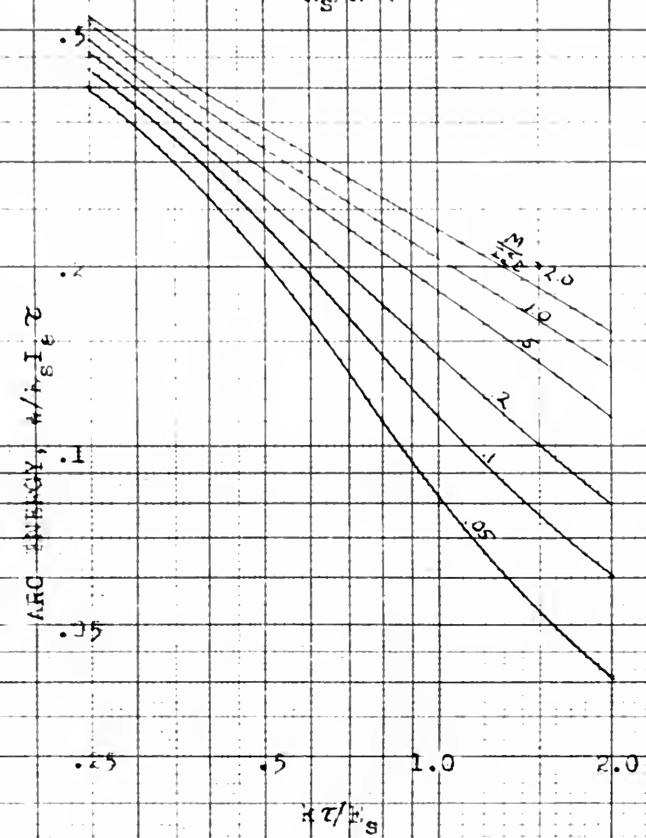
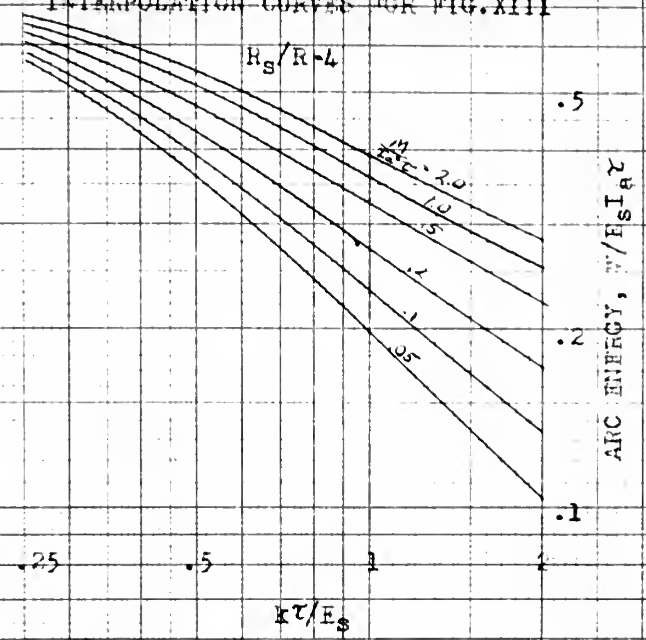


FIGURE XIV D

INTERPOLATION CURVES FOR FIG. XIII

$R_s/R = 4$



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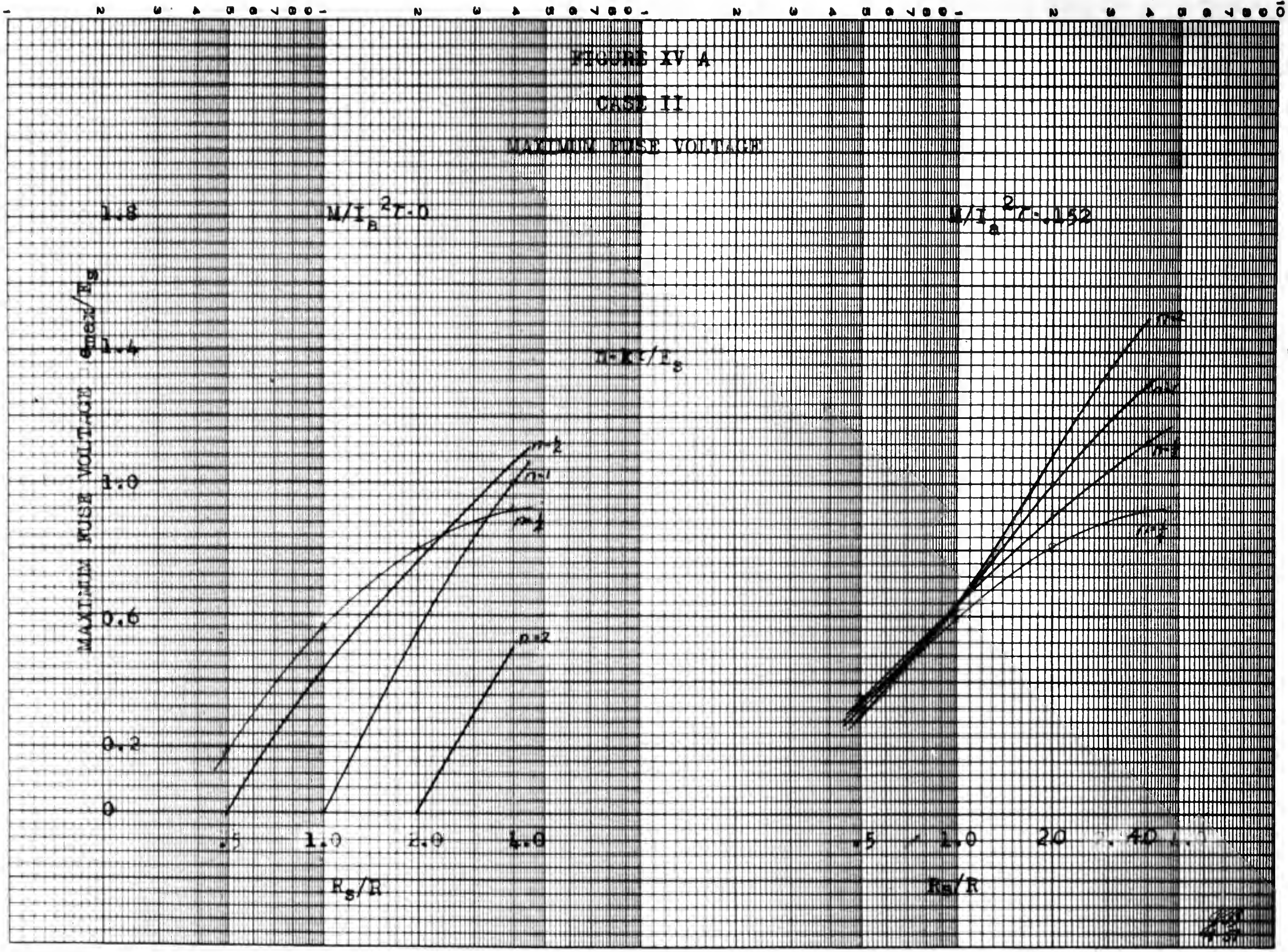


FIGURE XV A
CASE II
MAXIMUM FUSE VOLTAGE
$M/I_s^2\tau = 0$
$M/I_s^2\tau = .152$
MAXIMUM FUSE VOLTAGE e_{max}/E_s
1.8
1.4
1.0
0.6
0.2
0
R_s/R
.5
1.0
2.0
4.0
R_s/R
.5
1.0
20
40

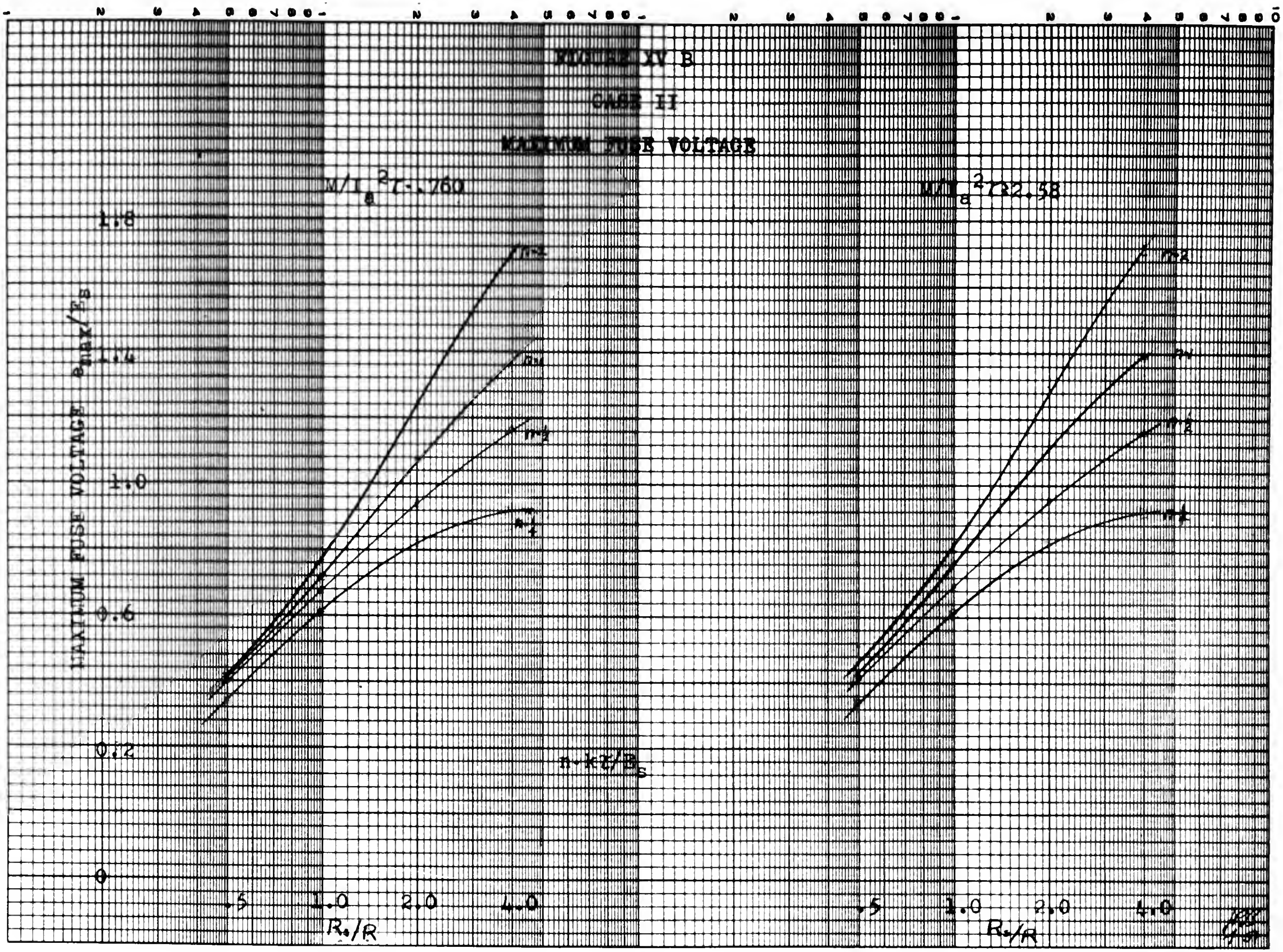


FIGURE XV B
CASE II
MAXIMUM FUSE VOLTAGE
M/I_a^2 τ = .760
M/I_a^2 τ 2.58
MAXIMUM FUSE VOLTAGE e_max/E_s
1.8
1.4
1.0
0.6
0.2
0
n = kτ/E_s
.5
1.0
2.0
4.0
R_s/R

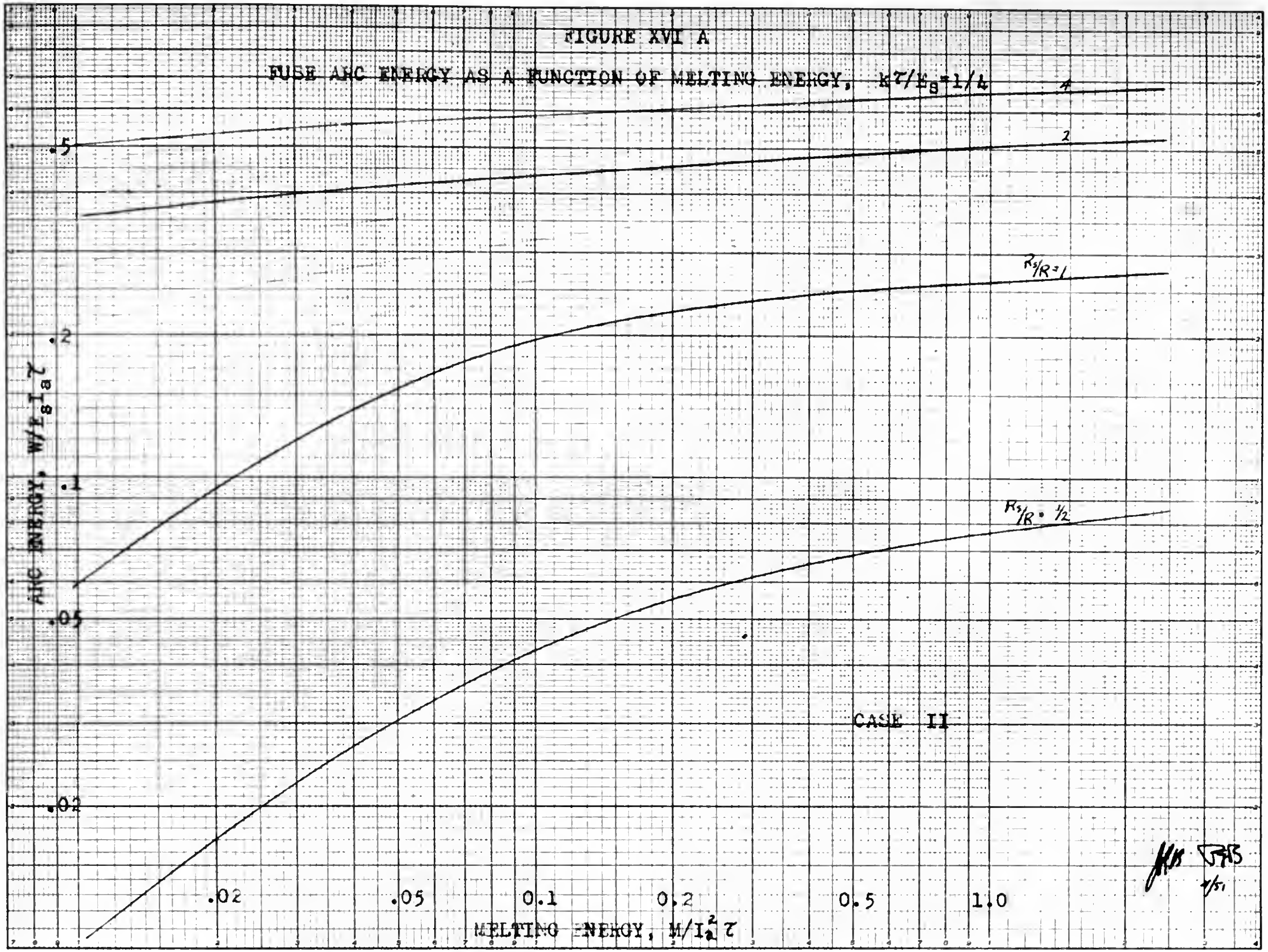


FIGURE XVI A

FUSE ARC ENERGY AS A FUNCTION OF MELTING ENERGY, $k\tau/E_s = 1/4$

FIGURE XVI B

CASE II

FUSE ARC ENERGY AS A FUNCTION OF MELTING ENERGY, $K'/E_s = 1/2$

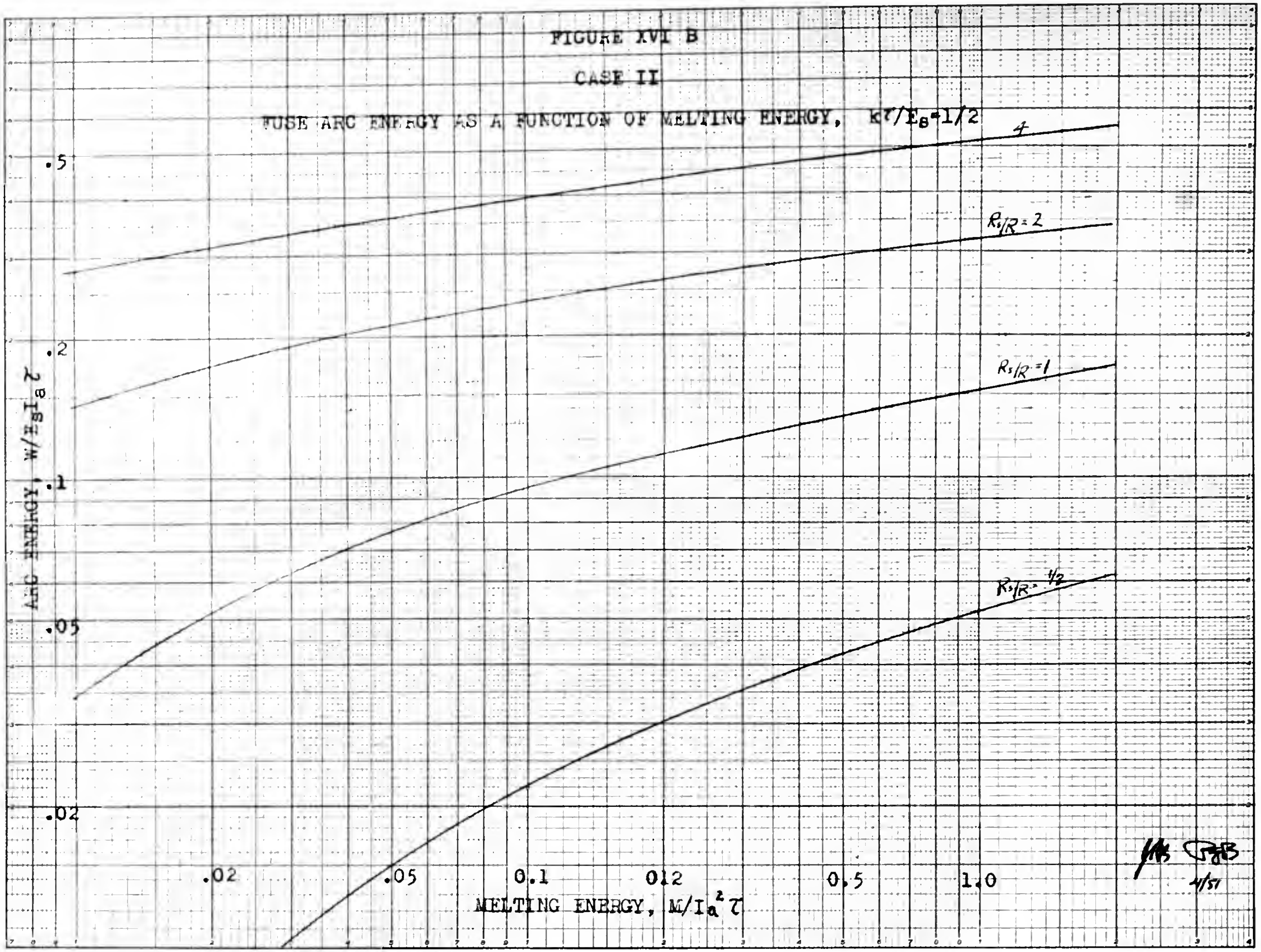
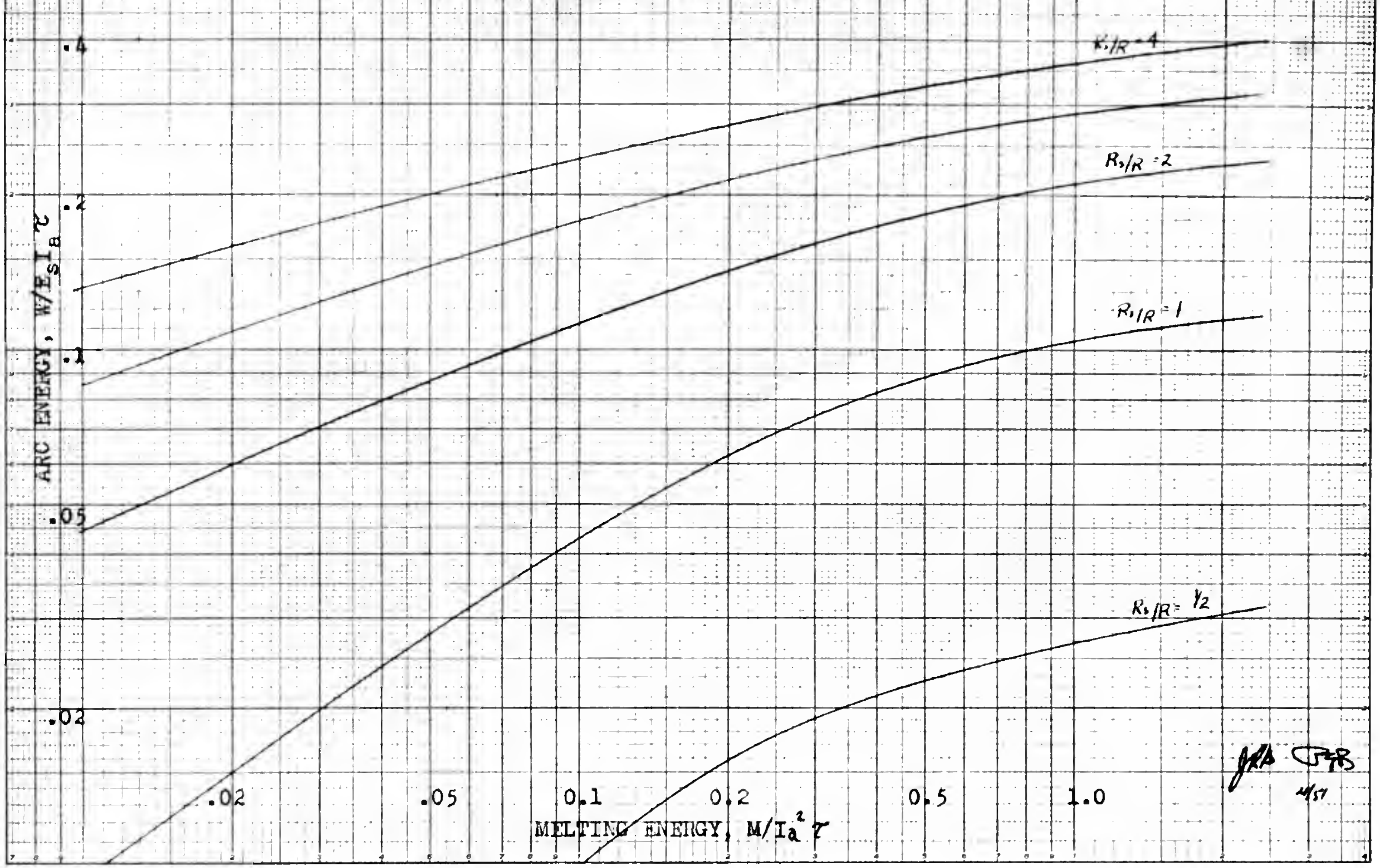


FIGURE XVI C

CASE II

FUSE ARC ENERGY AS A FUNCTION OF MELTING ENERGY, $k\tau/E_s = 1$



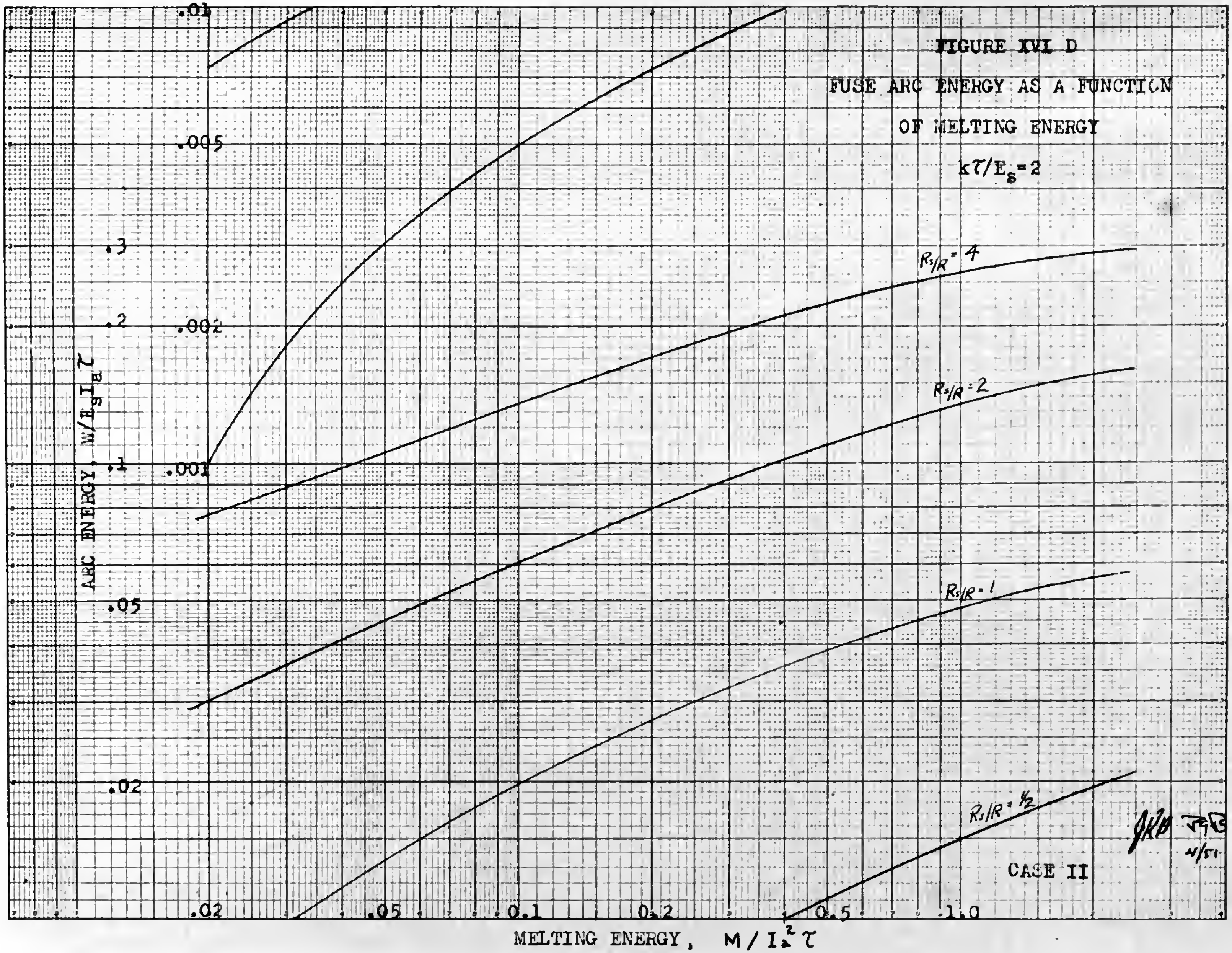


FIGURE XVI D
FUSE ARC ENERGY AS A FUNCTION
OF MELTING ENERGY
$k\tau/E_s = 2$
$R_s/R = 4$
$R_s/R = 2$
$R_s/R = 1$
$R_s/R = 1/2$
CASE II
ARC ENERGY, $W/E_s I_a \tau$
MELTING ENERGY, $M/I_a^2 \tau$
.01
.005
.002
.001
.3
.2
.1
.05
.02
.02
.05
0.1
0.2
0.5
1.0
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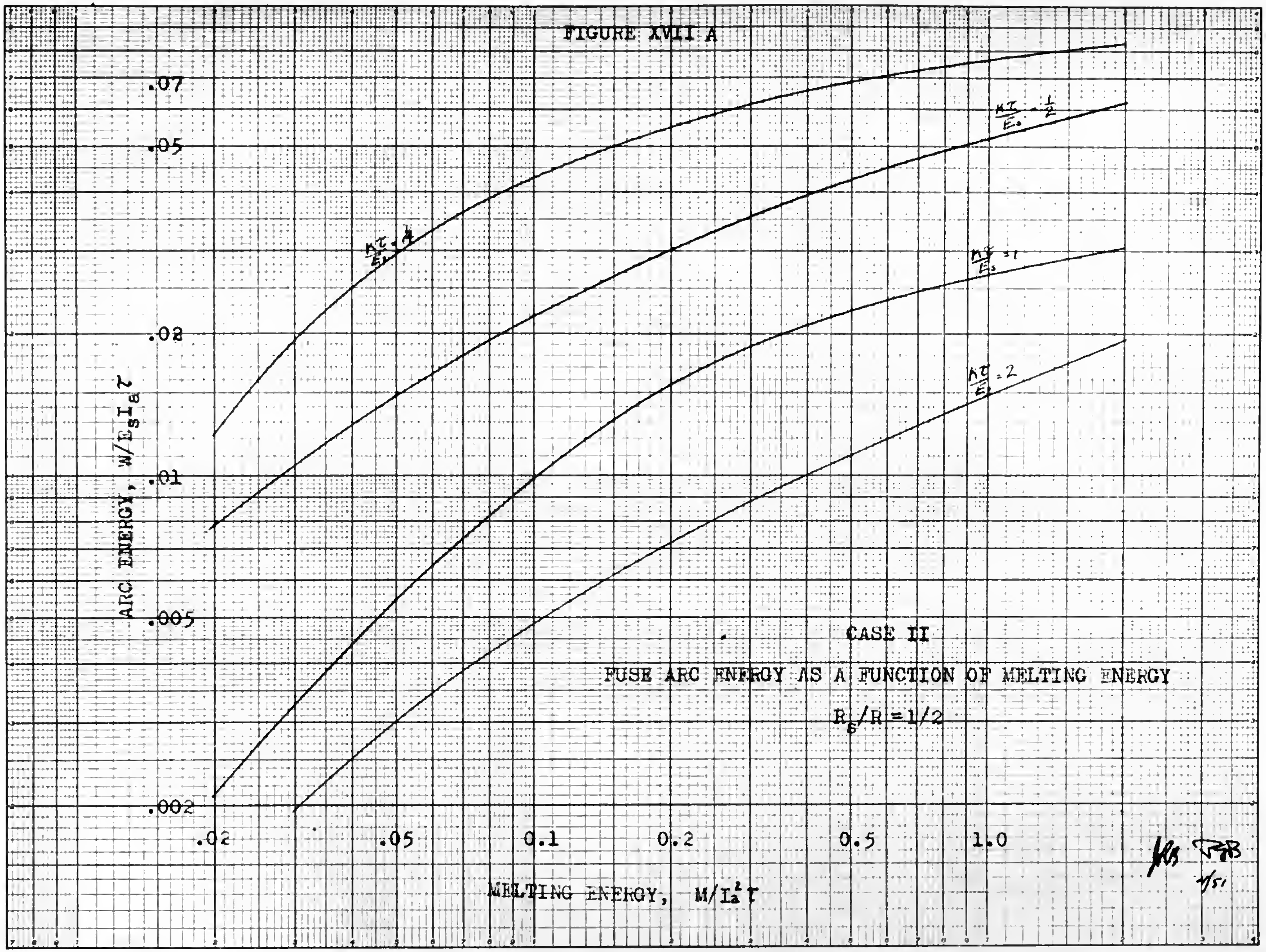


FIGURE XVII A
CASE II
FUSE ARC ENERGY AS A FUNCTION OF MELTING ENERGY
$R_s/R = 1/2$
ARC ENERGY, $W/E_sI_s\tau$
MELTING ENERGY, $M/I_a^2\tau$
.07
.05
.02
.01
.005
.002
.02
.05
0.1
0.2
0.5
1.0
$\frac{h\tau}{E_s} = \frac{1}{4}$
$\frac{h\tau}{E_s} = \frac{1}{2}$
$\frac{h\tau}{E_s} = 1$
$\frac{h\tau}{E_s} = 2$

FIGURE XVIII B

CASE II

FUSE ARC ENERGY AS A FUNCTION OF MELTING ENERGY, $H_s/H = 1$

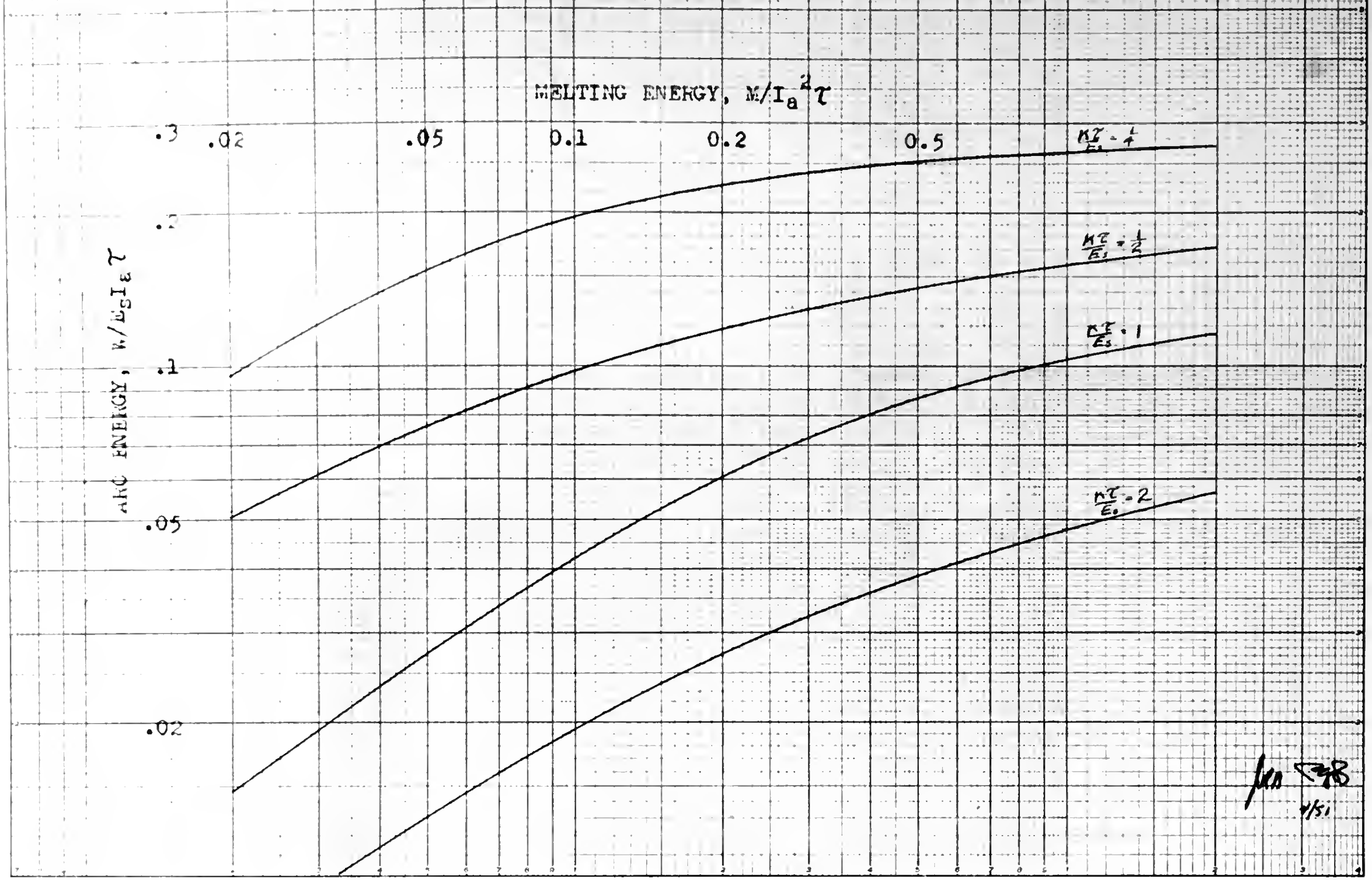


FIGURE XVII C

CASE II

FUSE ARC ENERGY AS A FUNCTION OF MELTING ENERGY, $R_s/R=2$

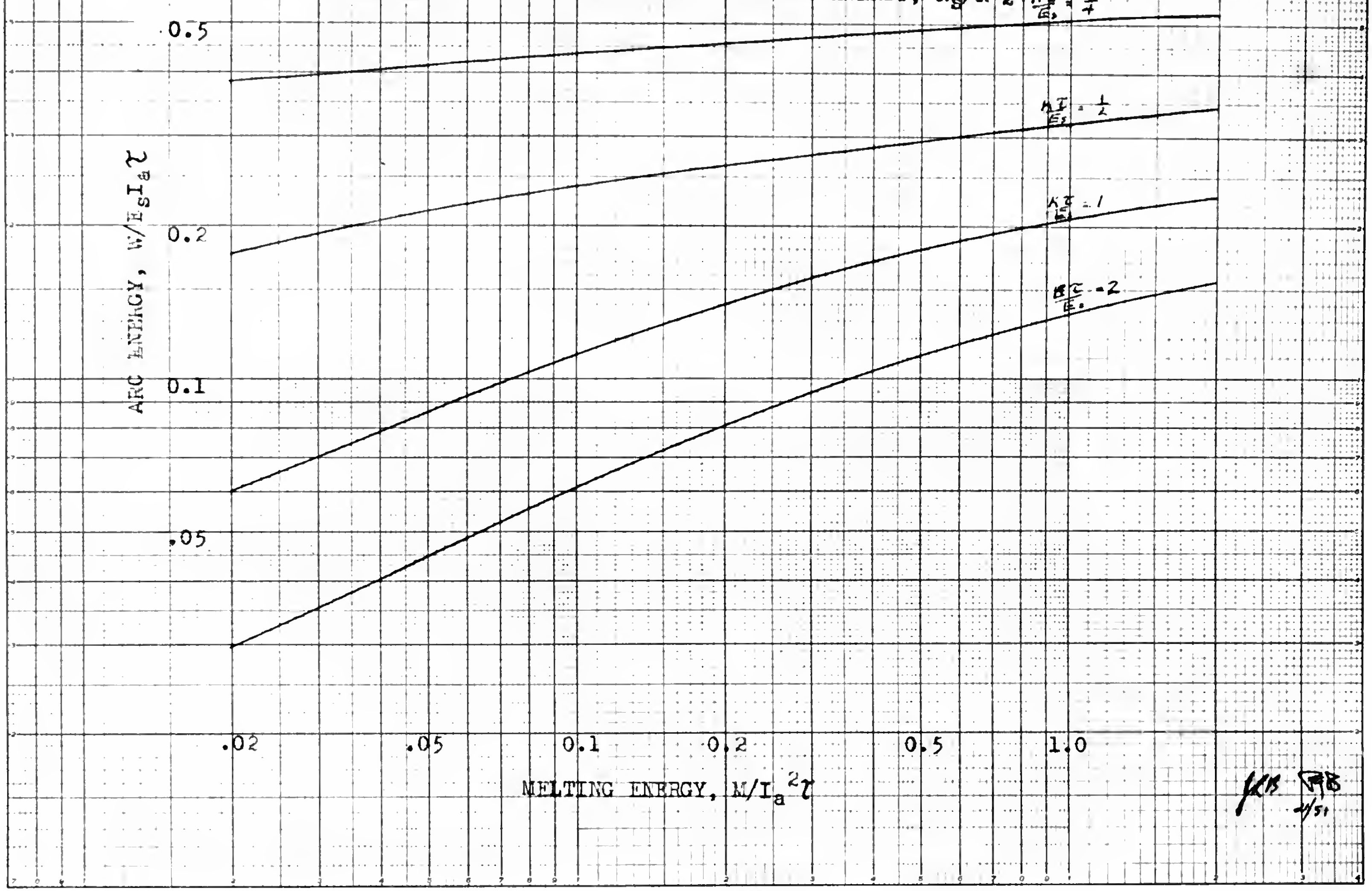
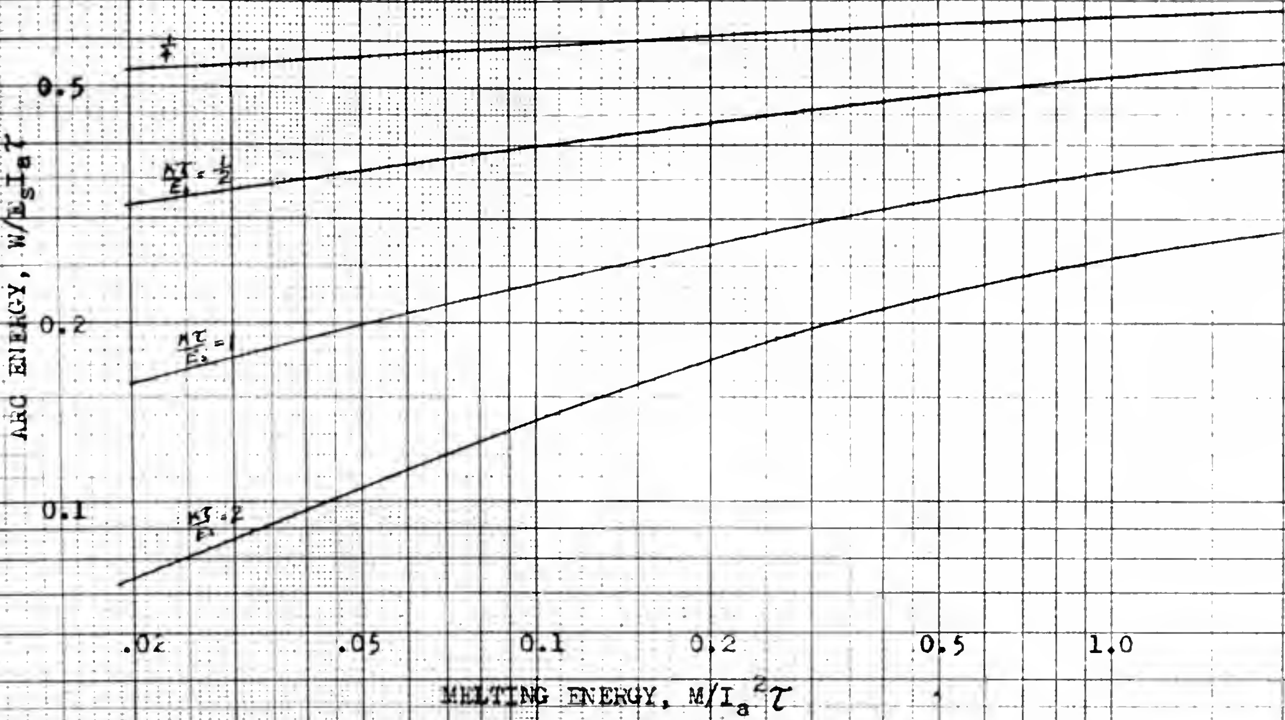


FIGURE XVII D



CASE II

FUSE ARC ENERGY AS A FUNCTION OF MELTING ENERGY

$R_s/R = 4$

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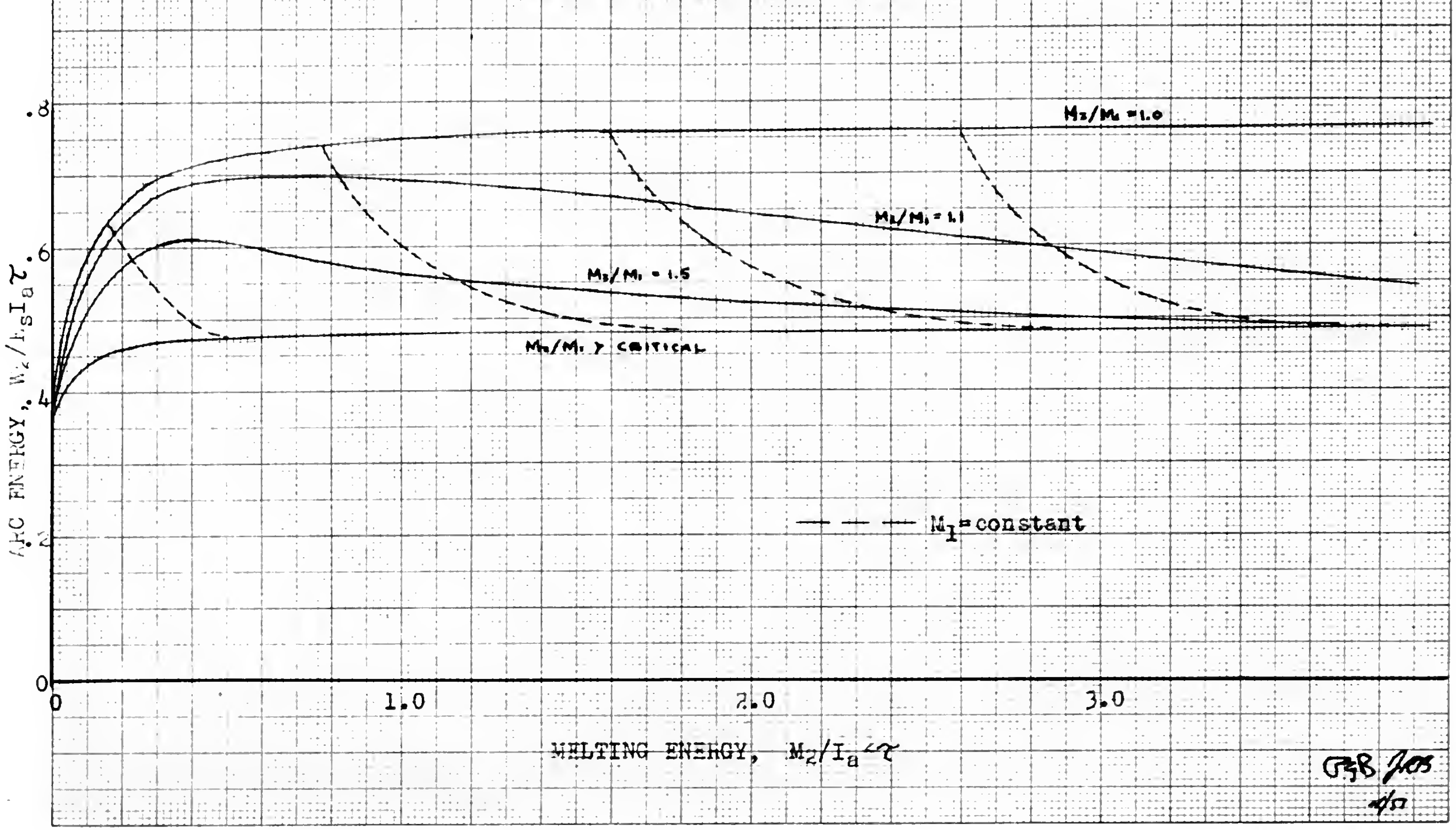
FIGURE XVIII A

CASE IV

EFFECT OF EARLY MELTING OF FUSE 2 ON FUSE 2 ARC ENERGY

$k_1\tau/I_s = 1/2,\ E_s/E = 1/2,\ k_2\tau/E_s = 1/2$



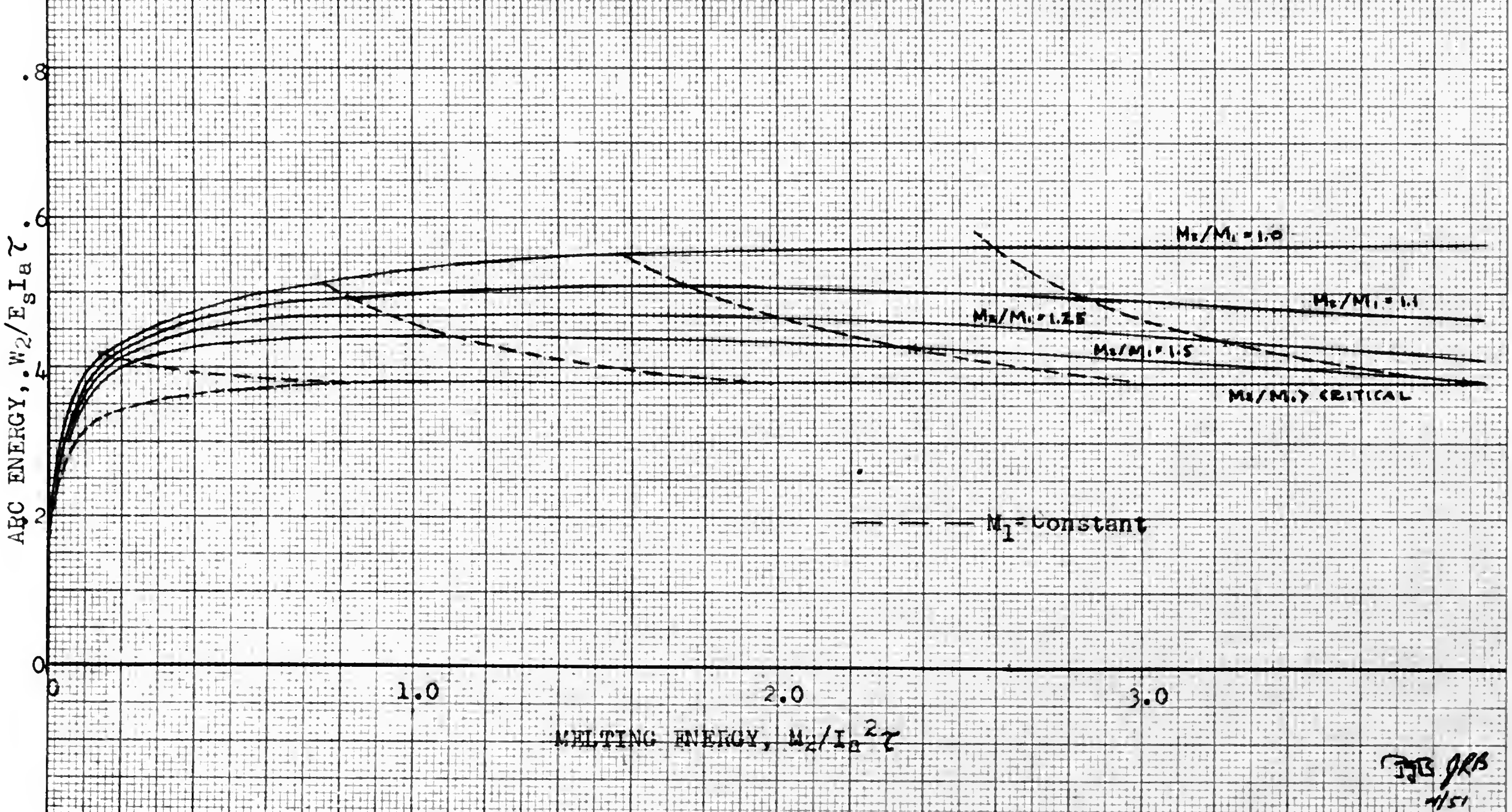


FIGURE XVIII B

CASE IV

EFFECT OF EARLY MELTING OF FUSE 2 ON FUSE 2 ARC ENERGY

$k_1\tau/E_s = 1/2,\ R_s/R = 1/2,\ k_2\tau/E_s = 1$

FIGURE XVIII C

EFFECT OF EARLY MELTING OF FUSE 2 ON FUSE 2 ARC ENERGY

$k_1\tau/E_s = 1/2, \quad R_s/R = 1/2, \quad k_2\tau/E_s = 2$

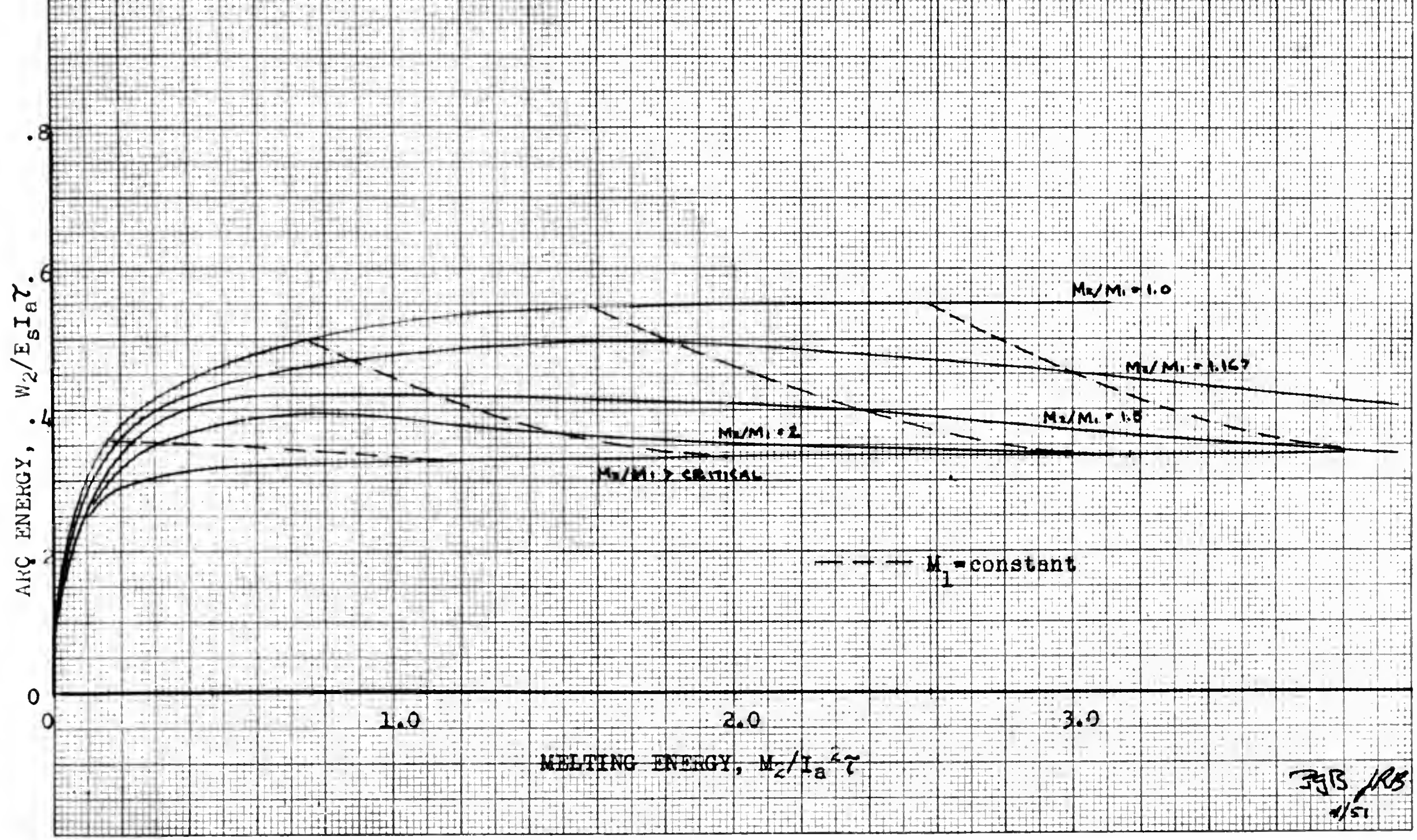


FIGURE XIX A

EFFECT OF EARLY MELTING OF FUSE 2 ON FUSE 2 ARC ENERGY

CASE IV

$k_1\tau/E_s = 1,\ R_s/R = 1/2,\ k_2\tau/E_s = 1/2$

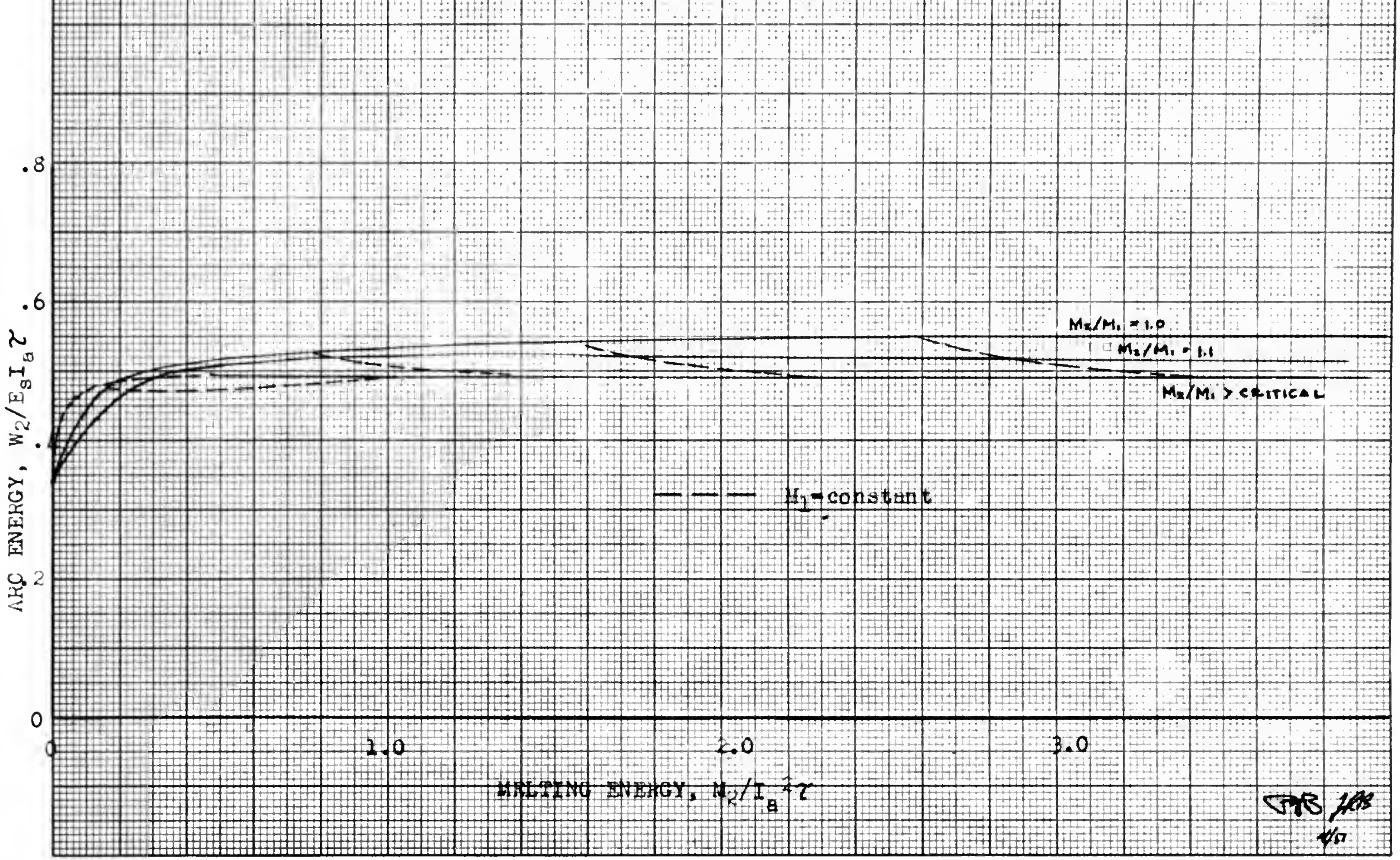
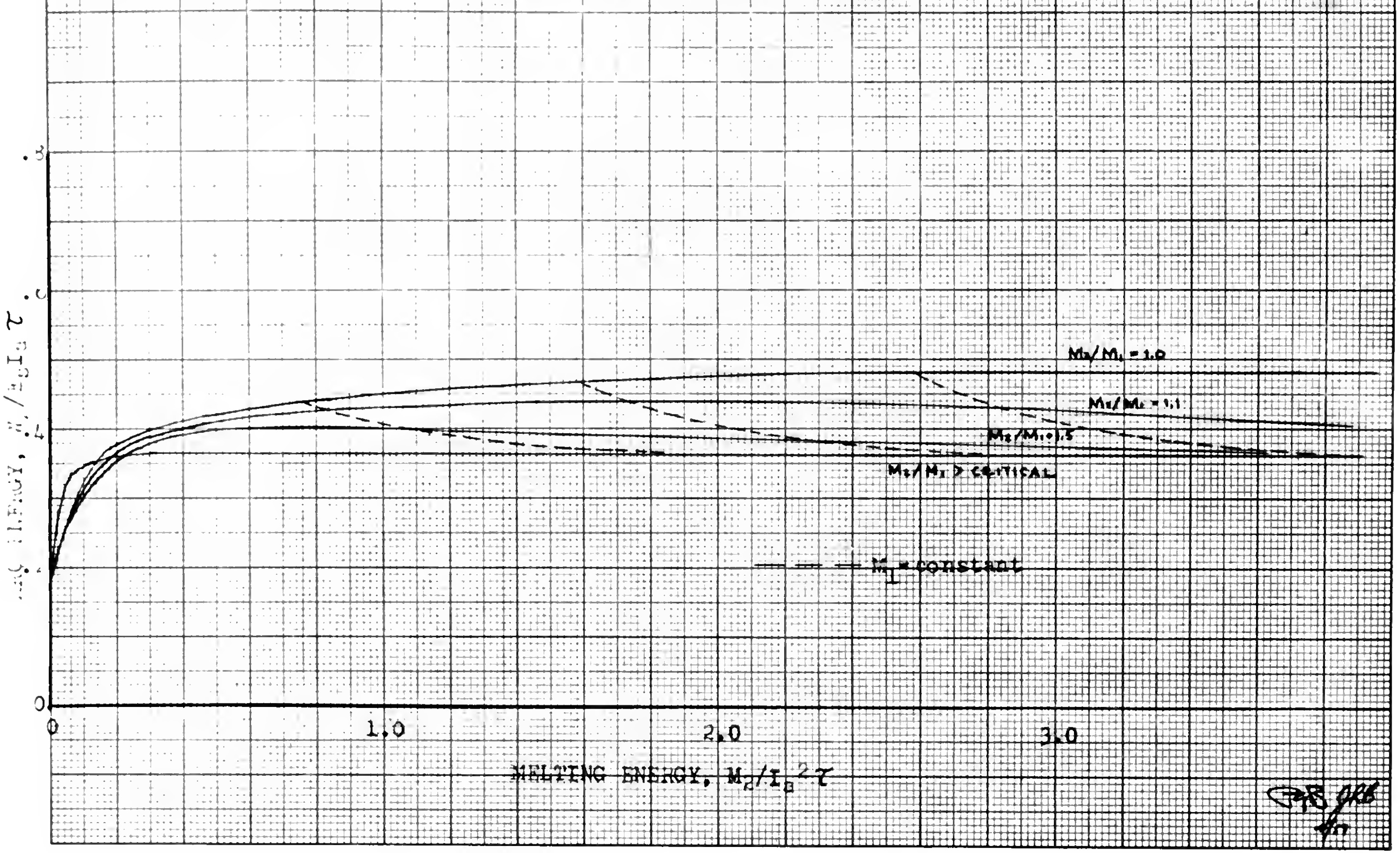


FIGURE XIX B

CASE IV

EFFECT OF EARLY MELTING OF FUSE 2 ON FUSE 2 ARC ENERGY

$k_1\tau/E_S = 1, \quad k_S/k = 1/2, \quad k_2\tau/E_S = 1$



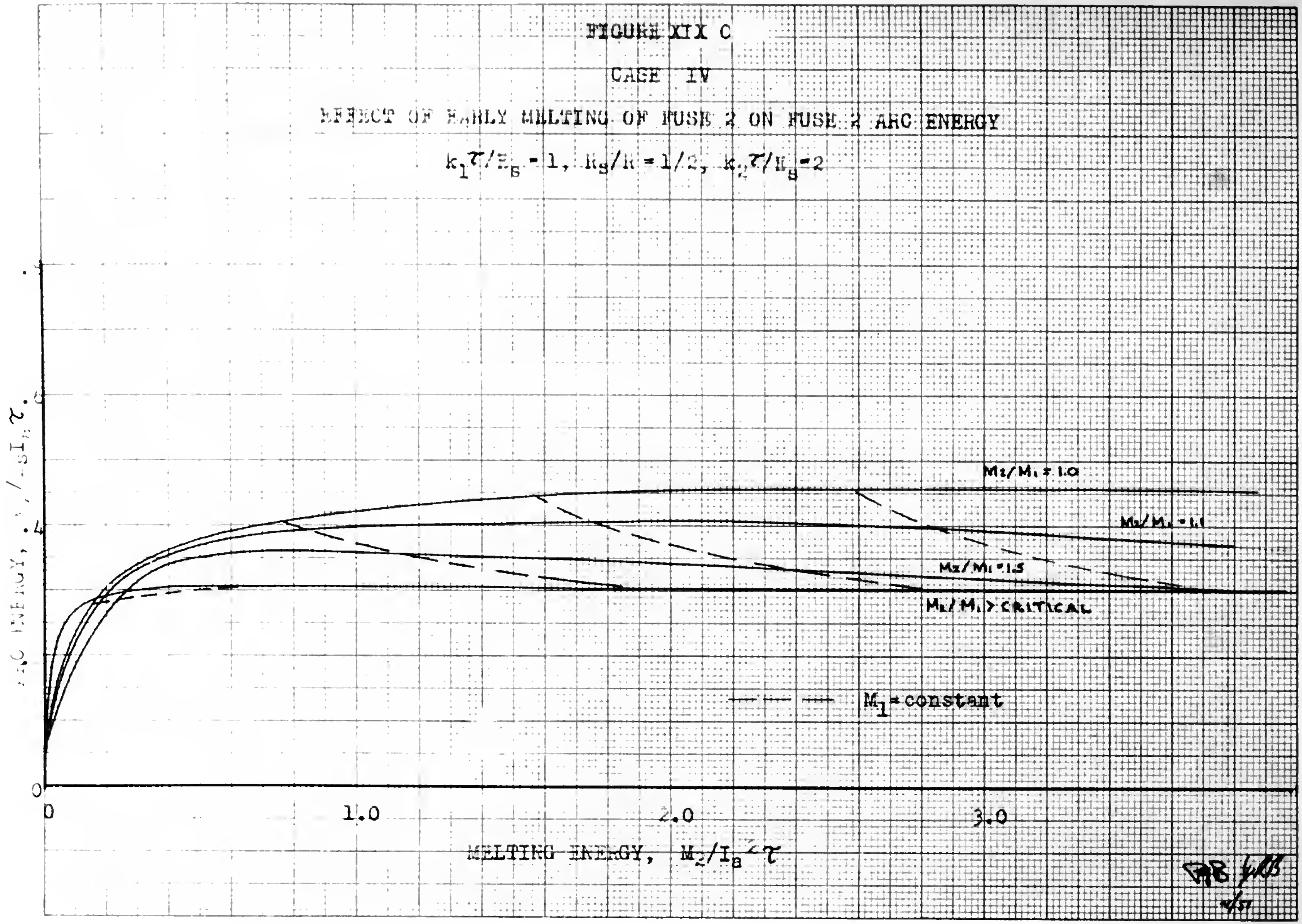


FIGURE XIX C
CASE IV
EFFECT OF EARLY MELTING OF FUSE 2 ON FUSE 2 ARC ENERGY
$k_1\tau/E_s = 1$, $R_s/R = 1/2$, $k_2\tau/E_s = 2$
$M_2/M_1 = 1.0$
$M_2/M_1 = 1.1$
$M_2/M_1 = 1.5$
$M_2/M_1 >$ CRITICAL
M_1 = constant
0
1.0
2.0
3.0
MELTING ENERGY, $M_2/I_s^2\tau$
.4
ARC ENERGY

FIGURE XX A

CASE IV

EFFECT OF EARLY MELTING OF FUSE 2 ON FUSE 2 ARC ENERGY

$k_1\tau/R_s=1,\ R_B/R=1,\ k_2\tau/R_B=1/4$

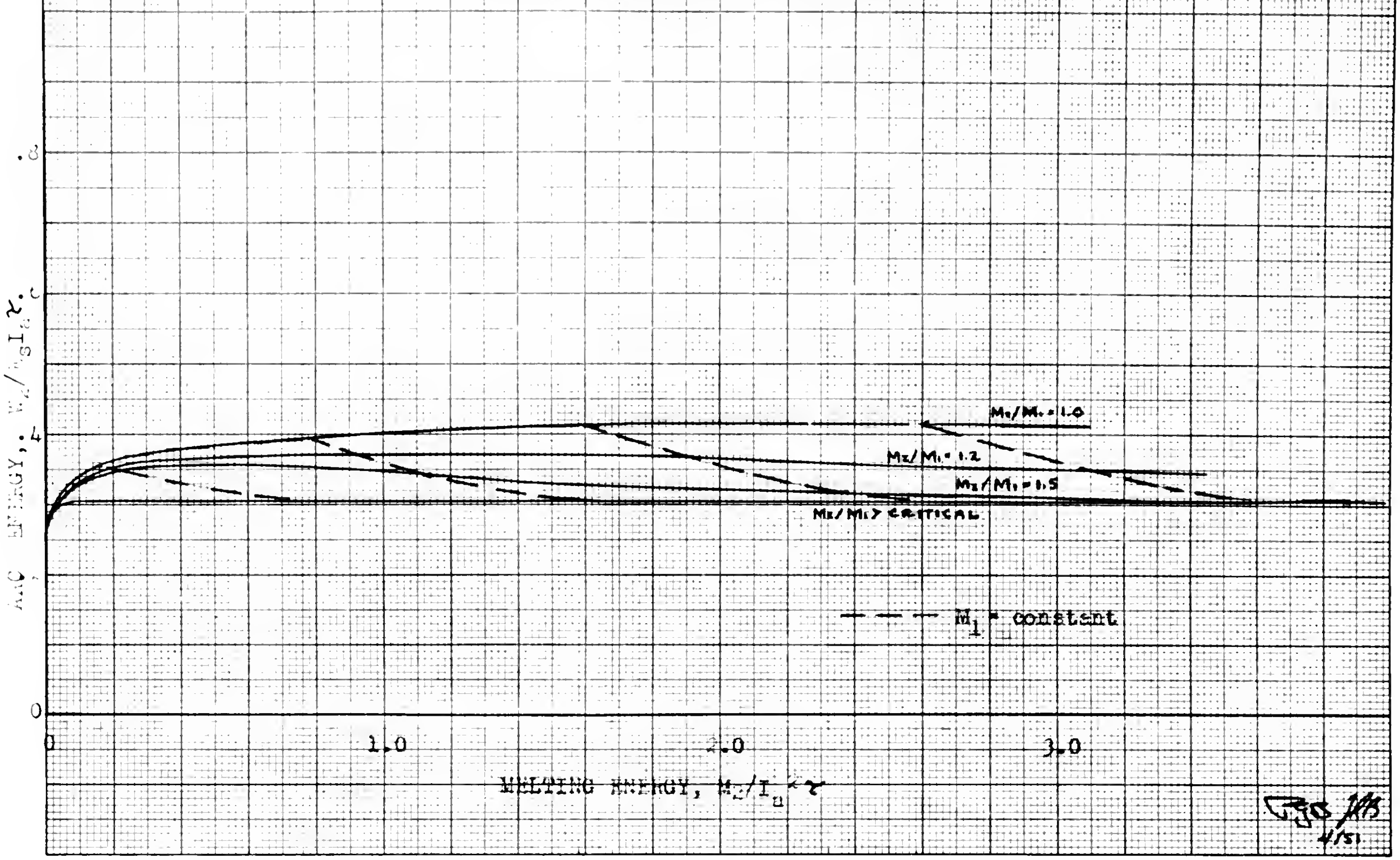


FIGURE XX B

CASE IV

EFFECT OF EARLY MELTING OF FUSE 2 ON FUSE 2 ARC ENERGY

$k_1\tau/E_s = 1,\ R_s/R = 1,\ k_2\tau/E_s = 1.$

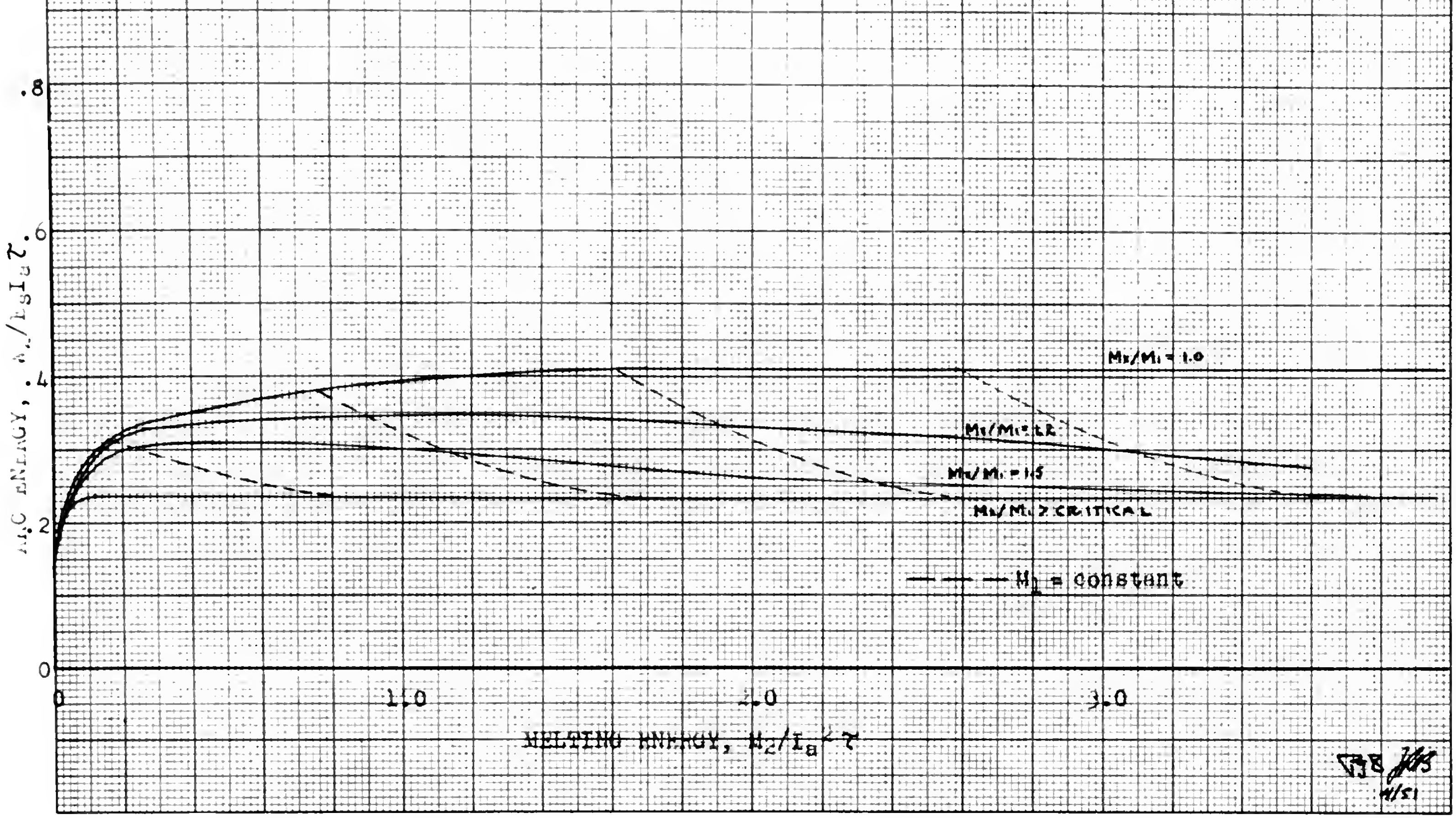


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FIGURE XX C

EFFECT OF EARLY MELTING OF FUSE 2 ON FUSE 2 ARC ENERGY

$k_1 \tau / E_s = 1,\ R_s/R = 1,\ k_2 \tau / E_s = 2$

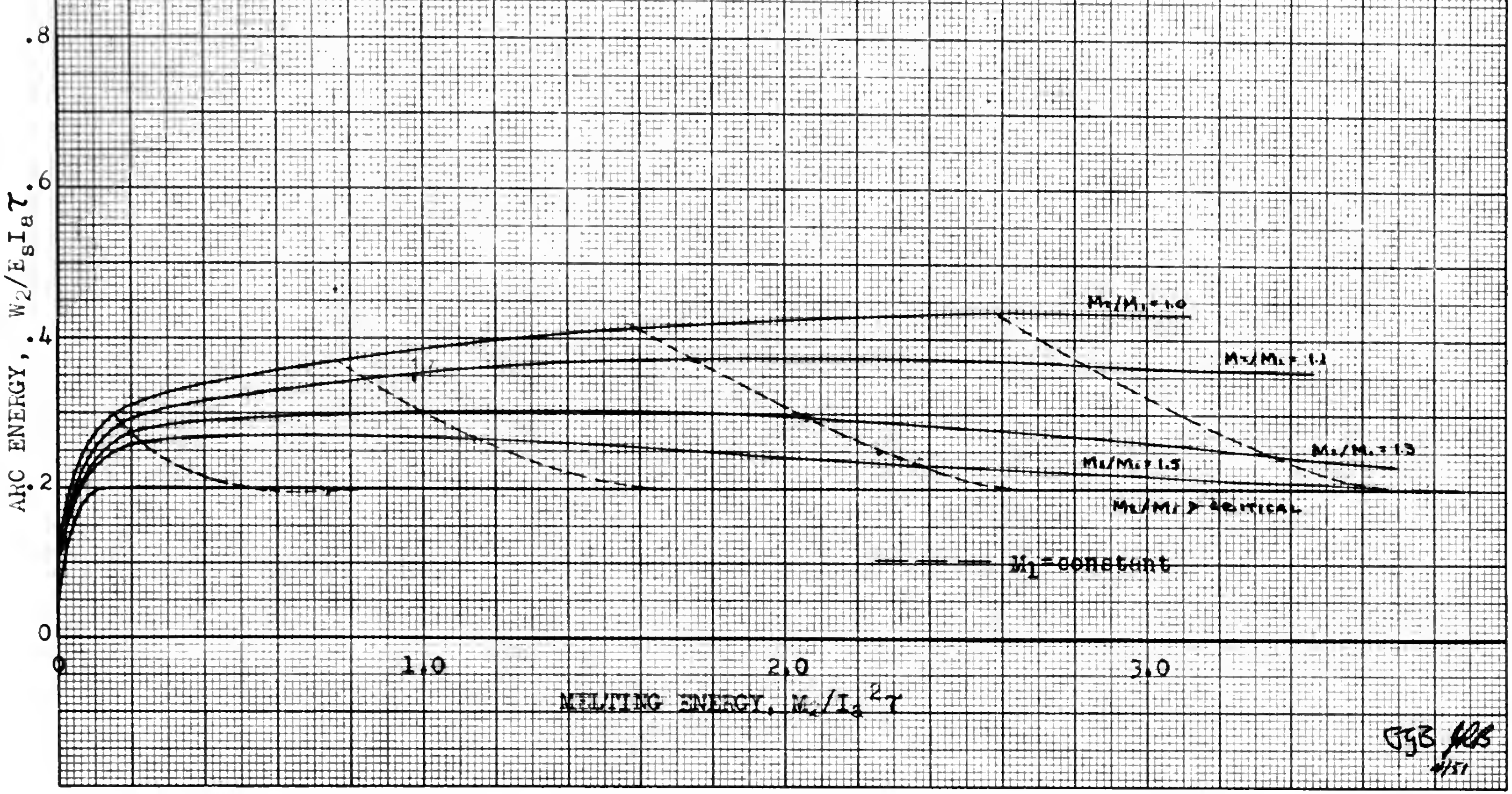


TABLE I

CRITICAL MELTING ENERGIES FOR FUSE 2, CASE IV

$K_1\tau/E_s = 1/4$					
R_s/R	$M_1/I_a^2\tau$				
	0	.152	.760	1.58	2.58
1/2	.46	1.25	2.12	3.12	4.06
1	.46	1.22	2.19	3.14	4.13
2	.50	1.21	2.20	3.12	4.16
4	.49	1.22	2.14	3.09	4.09
	$K_1\tau/E_s = 1/2$				
1/2	.47	1.10	1.98	2.99	3.94
1	.31	1.06	1.79	2.82	3.82
2	.28	.95	1.86	2.83	3.80
4	.28	.94	1.86	2.84	3.76
	$K_1\tau/E_s = 1$				
1/2	.48	1.03	1.82	2.82	3.78
1	.33	.85	1.68	2.66	3.69
2	.18	.75	1.54	2.55	3.57
4	.11	.68	1.51	2.50	3.46
	$K_1\tau/E_s = 2$				
1/2	.47	1.05	1.80	2.82	3.71
1	.32	.79	1.58	2.49	3.54
2	.14	.55	1.40	2.32	3.27
4	.09	.50	1.28	2.18	3.17

DISCUSSION OF RESULTS

The purpose of this thesis is to lessen the problems confronting a fuse designer. The designer must consider the effects of the environment of the fuse. He is faced by conflicting requirements of a high rated current and a low allowable fault current. He is faced with the necessity of producing a fuse or limiter which will interrupt rapidly with a limited maximum voltage. The designer must produce a fuse or limiter which will not malfunction under a wide lattitude of fault conditions. This design problem is complicated by a lack of understanding of the dynamics of the arc. The investigation conducted here has been an attempt to put some method of comparison at the designer's disposal. The quantities considered were the total arc energy and the maximum voltage, since these offer a convenient way of estimating the difficulty of interruption, as noted in the introductory section. Figures VIII through XX present these two criteria for current limiters of several forms.

Assumptions

The assumptions used in arriving at the results are in general very simple. We have assumed that the fuse or limiter, can be represented by a pure voltage source having a linearly rising characteristic starting at the time of melting. This is a good approximation of the actual voltage produced by some limiters. It is not universally applicable; but it is generally much better where current is successfully limited to a minor fraction of that available, than for interruptions in which available current is closely approached. It was assumed that the melting energy can

be represented by a quantity having the dimensions of the product of current squared and time, and that this melting energy is constant for a given fuse. Finally, it was assumed that the circuit could be represented by a linear lumped parameter circuit containing only resistance and inductance.

Case I

Case I is the single fusable element limiter. Figure VIII contains contours of normalized arc energy as a function of melting energy and rate of rise of fuse arc voltage. To use figures of this type the designer must convert the dimensional melting energy, I^2t, to the normalized form $I^2t/I_a^2\tau$, and the dimensional rate of rise of arc voltage k, to the normalized form $k\tau/E_s$. Entering with these two quantities, the normalized arc energy can be found. To convert this to dimensional arc energy multiply by $E_sI_a\tau$. Figure IX is used similarly, multiplying the normalized voltage from the figure by the quantity E_s to find the maximum arc voltage. On these figures, varying any single parameter will generally result in moving across the contour surface in straight lines, making them particularly useful in studying the effect of changing one parameter at a time. Figures XI and XII show the same results in a different form which is useful whenever Figures VIII and IX are unwieldy. To examine the effect of varying the rate of rise of fuse arc voltage we can enter the contours for constant melting energy. Ordinarily the effect is readily seen and replotting the data is not necessary.

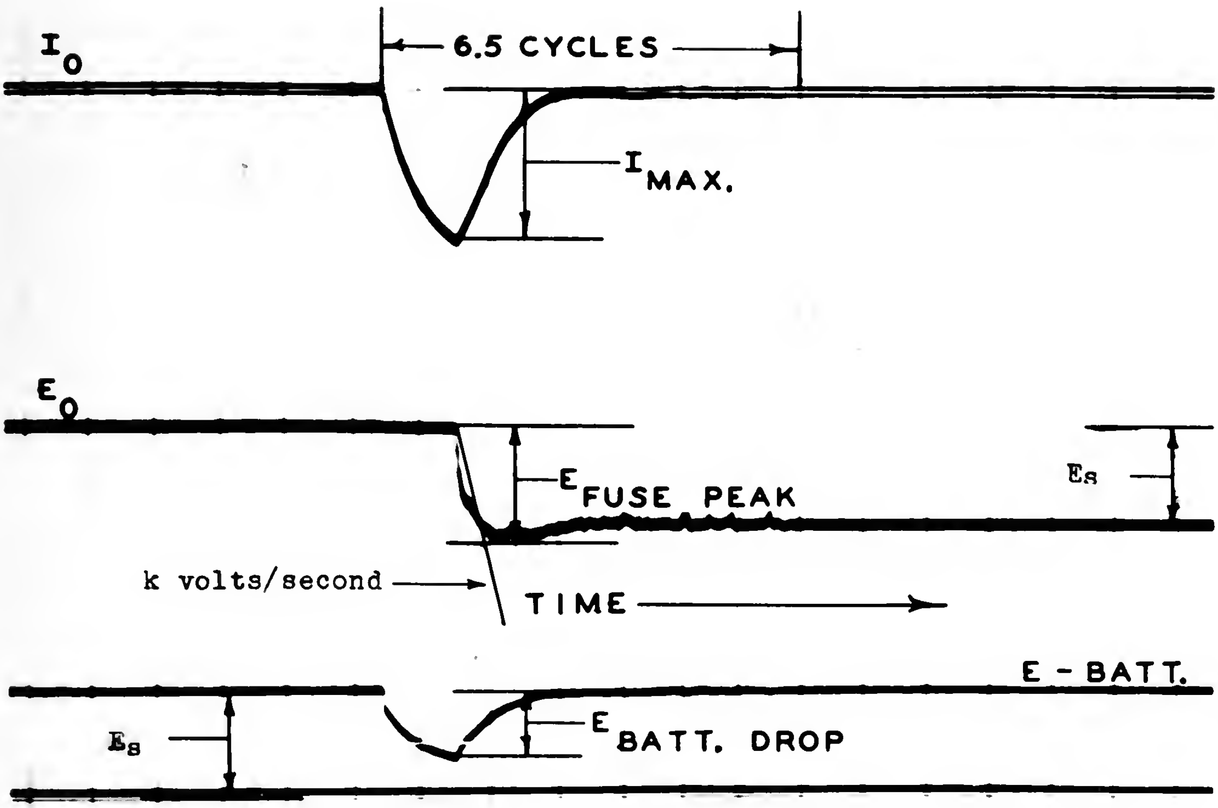
In using the energy and voltage contours some assumption must be made as to the way the rate of ruse of arc voltage varies. In design be represented by a quantity having the dimensions of the product of current squared and time, and that this melting energy is a constant for a given fuse. Finally, it was assumed that the circuit could be represented by a linear lumped parameter circuit containing only resistance and inductance.

Case I

Case I is the single fusable element limiter. Figure [illegible] contains contours of normalized arc [illegible] as a function of melting energy and rate of rise of fuse arc voltage. To use figures of this type the designer must convert the dimensional melting energy I^2t to the normalized form $I^2t/I_s^2\tau$ and the dimensional rate of rise of arc voltage [illegible] to the normalized form $k\tau/E_s$. Entering with these two quantities, the normalized [illegible] energy can be found. To convert this to dimensional arc energy multiply by $E_s I_s \tau$. Figure [illegible] is used similarly, multiplying the normalized voltage from the figure by the quantity E_s to find the maximum arc voltage. [illegible] these figures, varying any single parameter [illegible] [illegible] the contour surface in straight lines, making them [illegible] in studying the effect of changing one parameter [illegible]. Figures [illegible] and XII show the same results in a different form [illegible] useful whenever Figures VIII and IX [illegible]. [illegible] the effect of varying the rate of rise of fuse arc voltage [illegible] contours for constant melting energy. [illegible] the data is not [illegible].

In using [illegible] voltage [illegible] must be made as to the way the rate of rise of [illegible] voltage varies [illegible] design

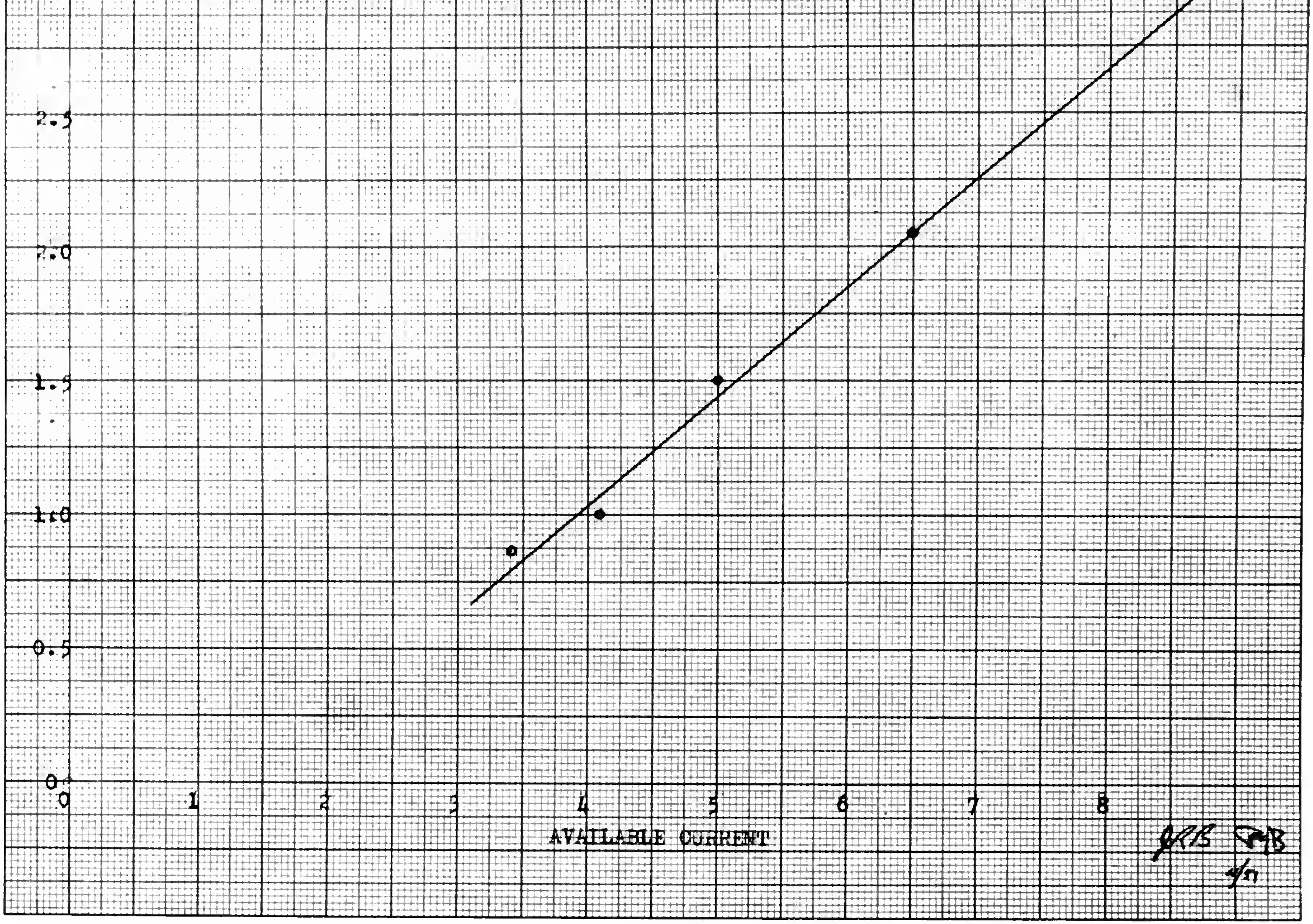
FIGURE XXI



METHOD OF DETERMINING AVARAGE RATE OF RISE OF ARC VOLTAGE

FIGURE XXII

AVERAGE RATE OF RISE OF FUSE VOLTAGE AS A FUNCTION OF AVAILABLE CURRENT



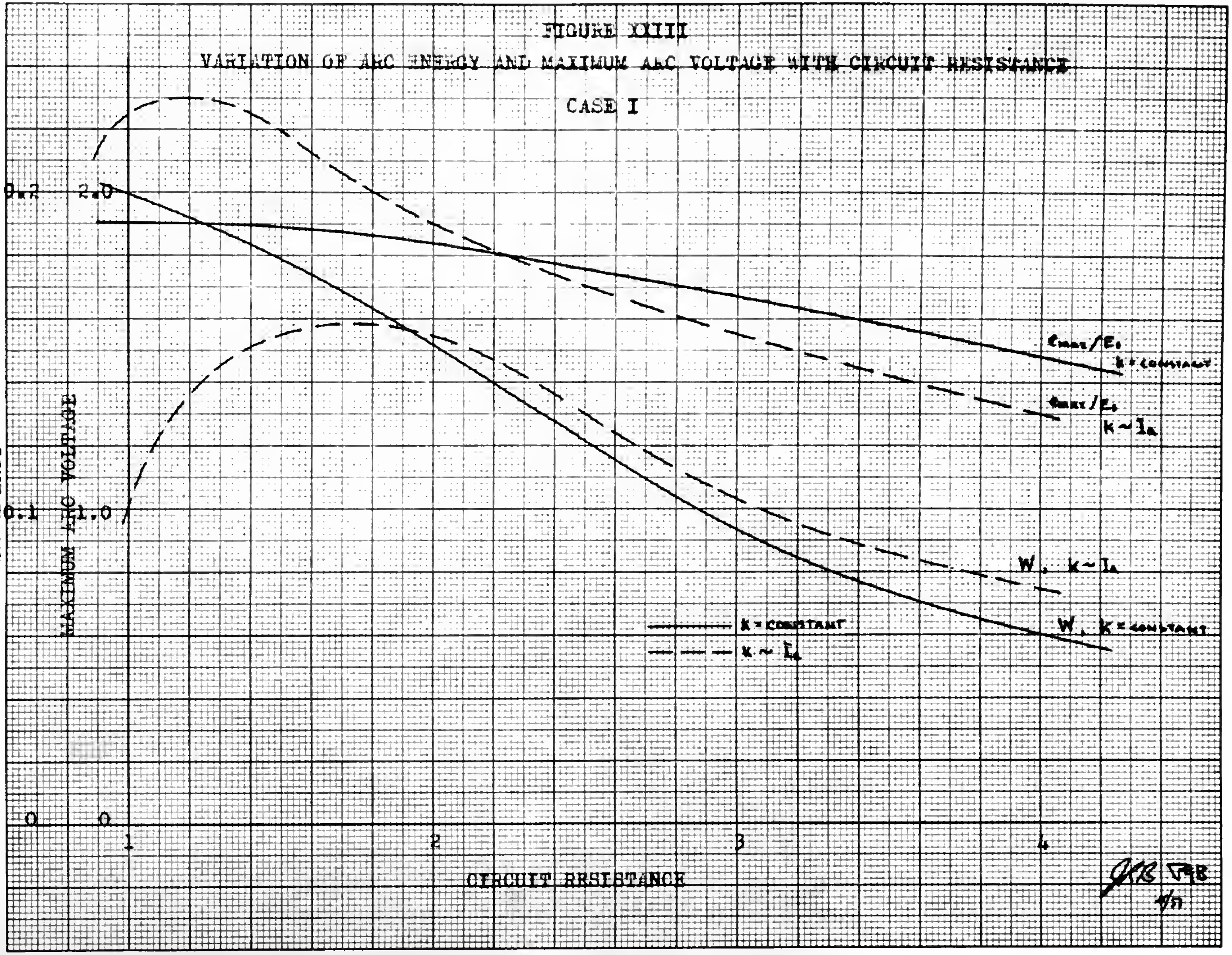


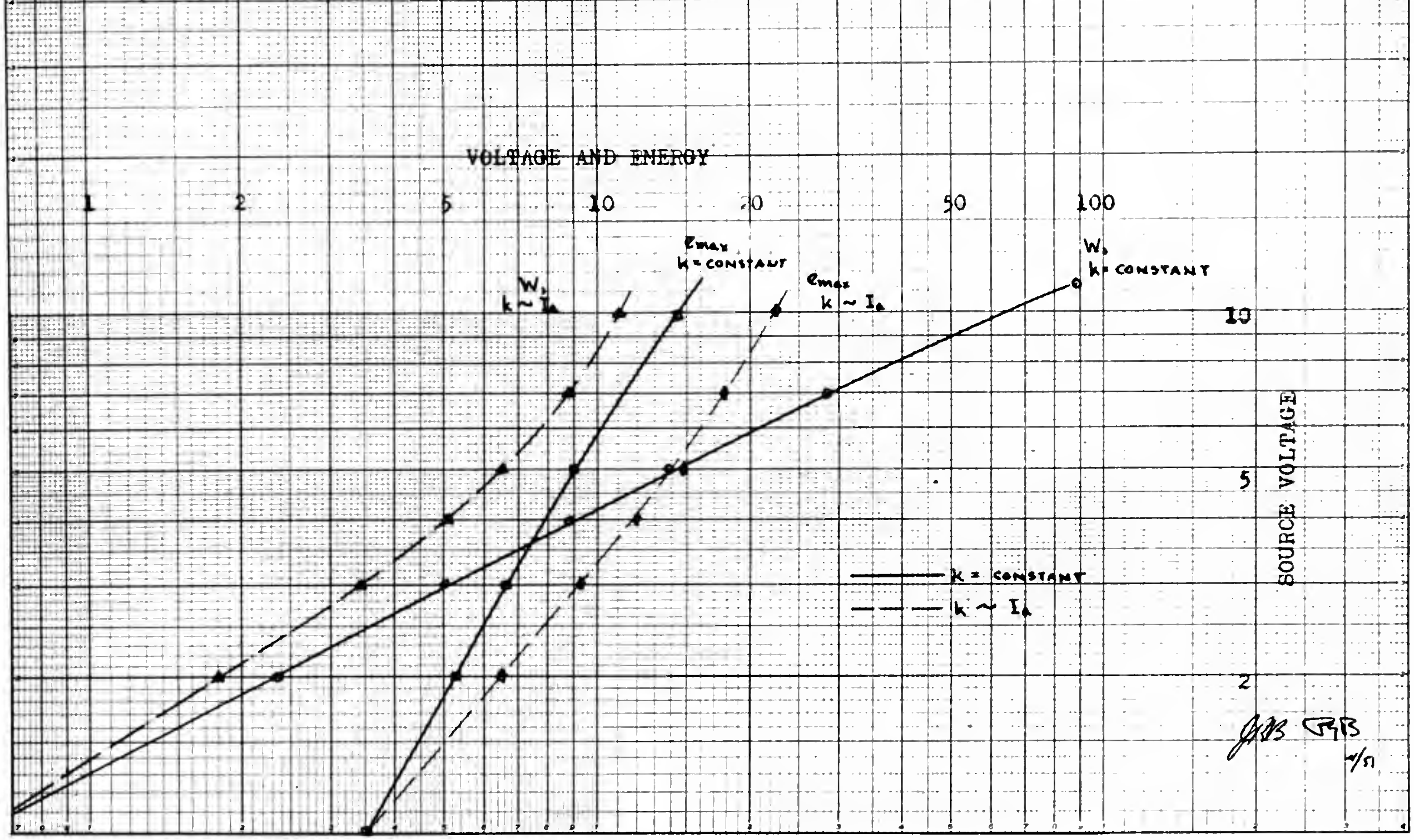
FIGURE XXIII

VARIATION OF ARC ENERGY AND MAXIMUM ARC VOLTAGE WITH CIRCUIT RESISTANCE

FIGURE XXIV

CASE I

VARIATION OF ARC ENERGY AND MAXIMUM ARC VOLTAGE WITH SOURCE VOLTAGE



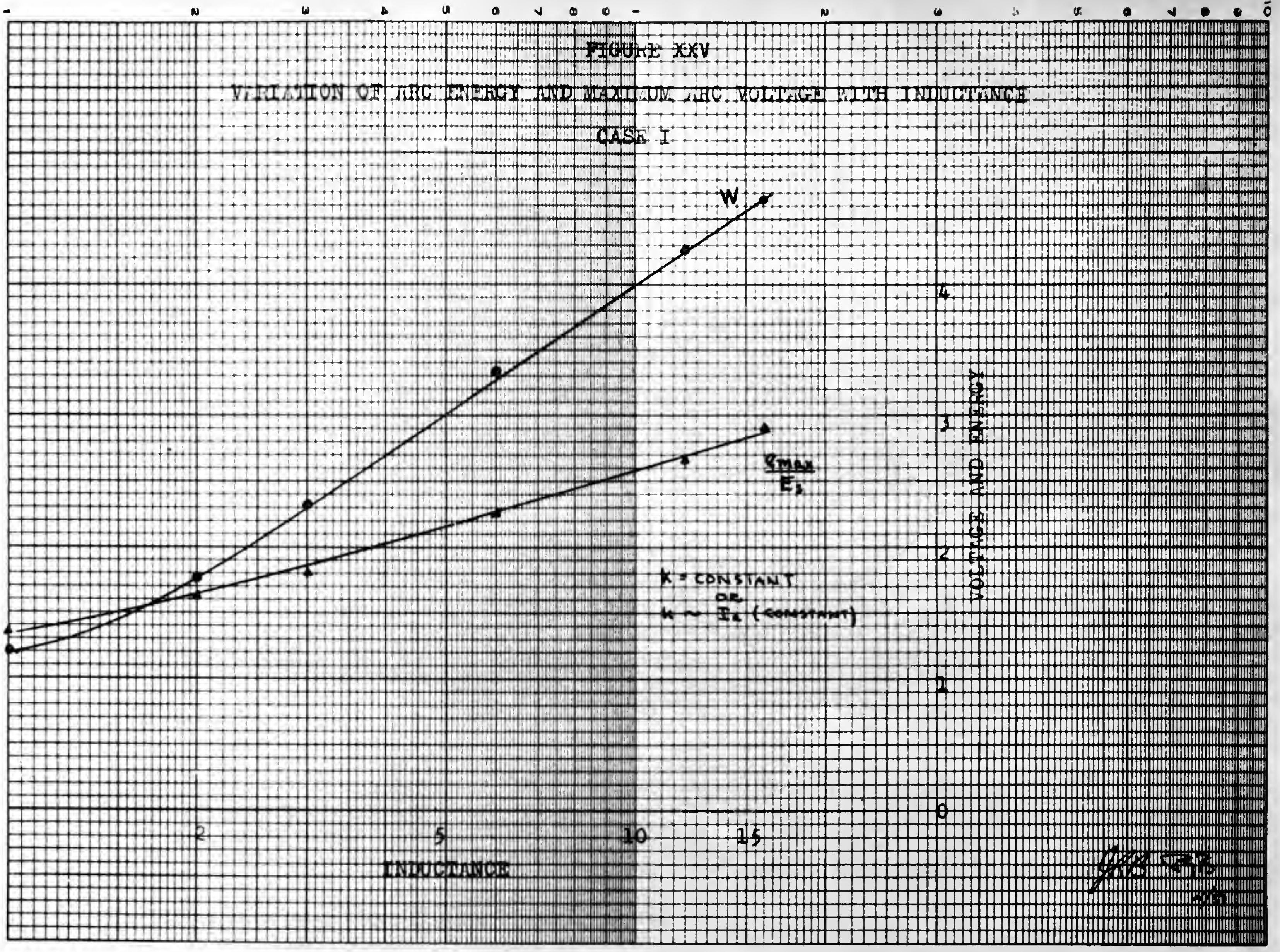


FIGURE XXV

VARIATION OF ARC ENERGY AND MAXIMUM ARC VOLTAGE WITH INDUCTANCE

CASE I

analysis this may be taken empirically from oscillograms of interruptions, as in Figure XXI. Figure XXII shows that in one limited test k can be assumed to vary as I_a, and this can be used to make a broad analysis of the performance of this limiter under varying conditions. In design synthesis, we would attempt to determine the optimum value of k for particular conditions, and so might study the effect of varying parameters with k held constant. The illustrations demonstrating the use of the results (Figures XXIII - XXV) were derived both for constant k and for k proportional to available current. This does not imply that these are the best ways to treat the rate of rise of arc voltage, but these are possible conditions.

Figure XXIII shows the variation of arc energy and maximum voltage with circuit resistance. Note that a study such as this shows the severest condition for a given fuse, circuit inductance, and source voltage. Figure XXIII was calculated for a hypothetical limiter and has direct significance only for that particular case. Such curves may be drawn from the contours for each fuse and each set of conditions. Figures XXIV and XXV are similar studies for variation of source voltage and for variation of circuit inductance. They are immediately useful in illustrating the use of the contours to evaluate the effect of varying one parameter at a time. In making the calculations for such a study, the effect of the variations on each of the entering quantities for the contours, and on the derived quantity must be considered.

The ability of a fuse to limit the magnitude of current after fault is of primary interest. Figure X shows the maximum current as a function of the melting energy and the slope of fuse arc voltage. From this the designer can determine the constants for design to meet given limiting requirements. This figure is valid for Case I and is also the maximum for all other cases provided the shunt resistance is also large enough to limit the current to the chosen maximum.

Case II

For Case II the limiter is a single fusable element shunted by a resistance. Figure XIII gives contours of constant arc energy as a function of shunt resistance and melting energy for particular values of arc voltage rate of rise. Figure XIV gives curves to be used for interpolation for values of rate of rise of voltage other than the slopes shown on Figure XIII. Together these figures define the solid map necessary to show the variation of arc energy with three parameters. The method of using these curves is the same as that described for Case I except for the additional variable R_s/R. They show that the arc energy increases with the value of the shunt resistance, approaching the value for Case I as a limit, at infinite R_s. They can be used to determine the effect of variations of single parameters just as contours for Case I were used.

Figures XVI and XVII present the information of Figure XIII in an alternate form. In some calculations these plots are easier to use. They show directly the effect of changing shunt resistance or the slope of arc voltage.

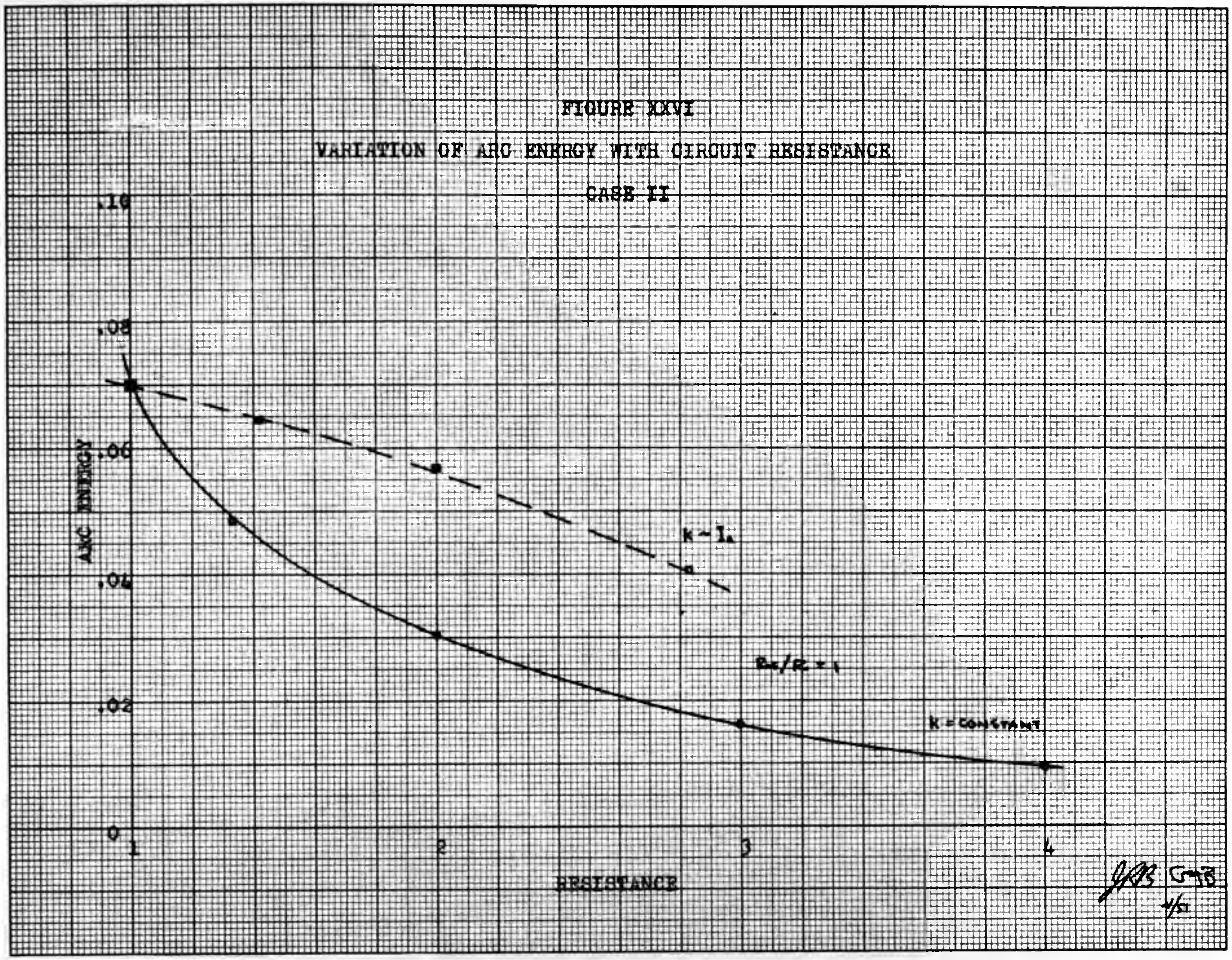


FIGURE XXVI

VARIATION OF ARC ENERGY WITH CIRCUIT RESISTANCE

CASE II

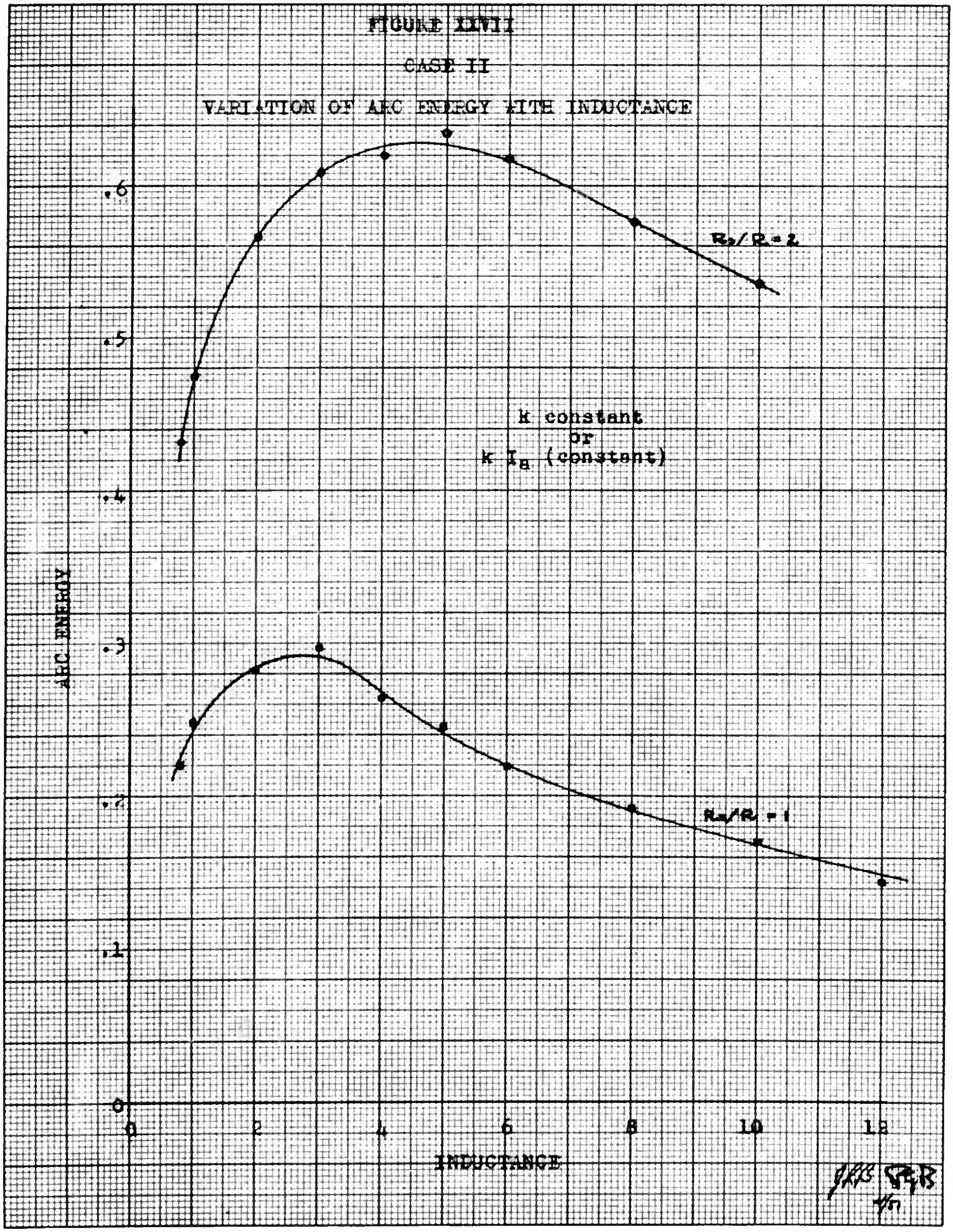


FIGURE XXVII
CASE II
VARIATION OF ARC ENERGY WITH INDUCTANCE
.6
.5
.4
.3
.2
.1
0
ARC ENERGY
0
2
4
6
8
10
12
INDUCTANCE
k constant
or
k I_a (constant)

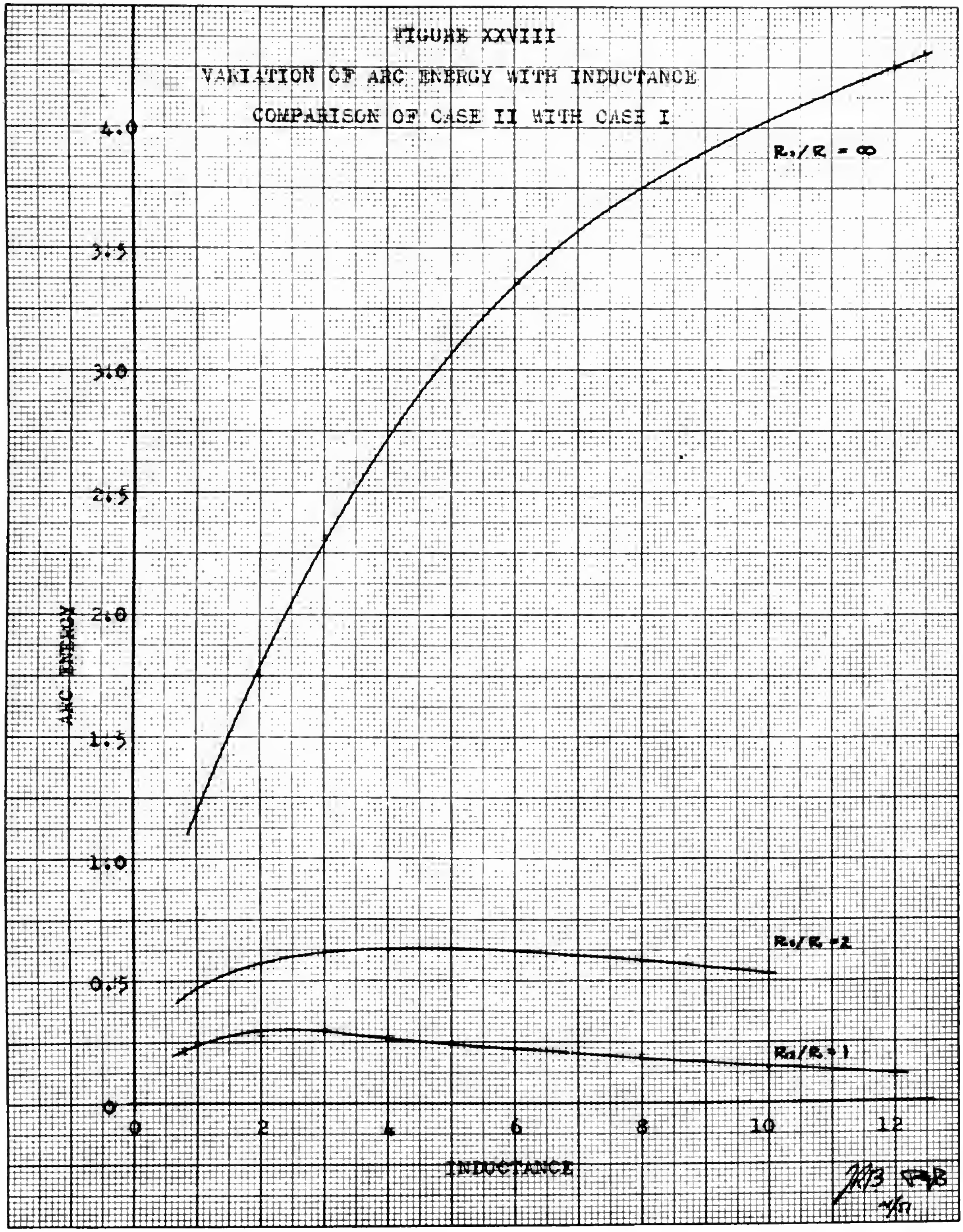


FIGURE XXVIII
VARIATION OF ARC ENERGY WITH INDUCTANCE
COMPARISON OF CASE II WITH CASE I
R₁/R = ∞
R₁/R₂ = 2
R₁/R₂ = 1
ARC ENERGY
INDUCTANCE
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
2
4
6
8
10
12

FIGURE XXIX

CASE II

VARIATION OF ARC ENERGY WITH SOURCE VOLTAGE

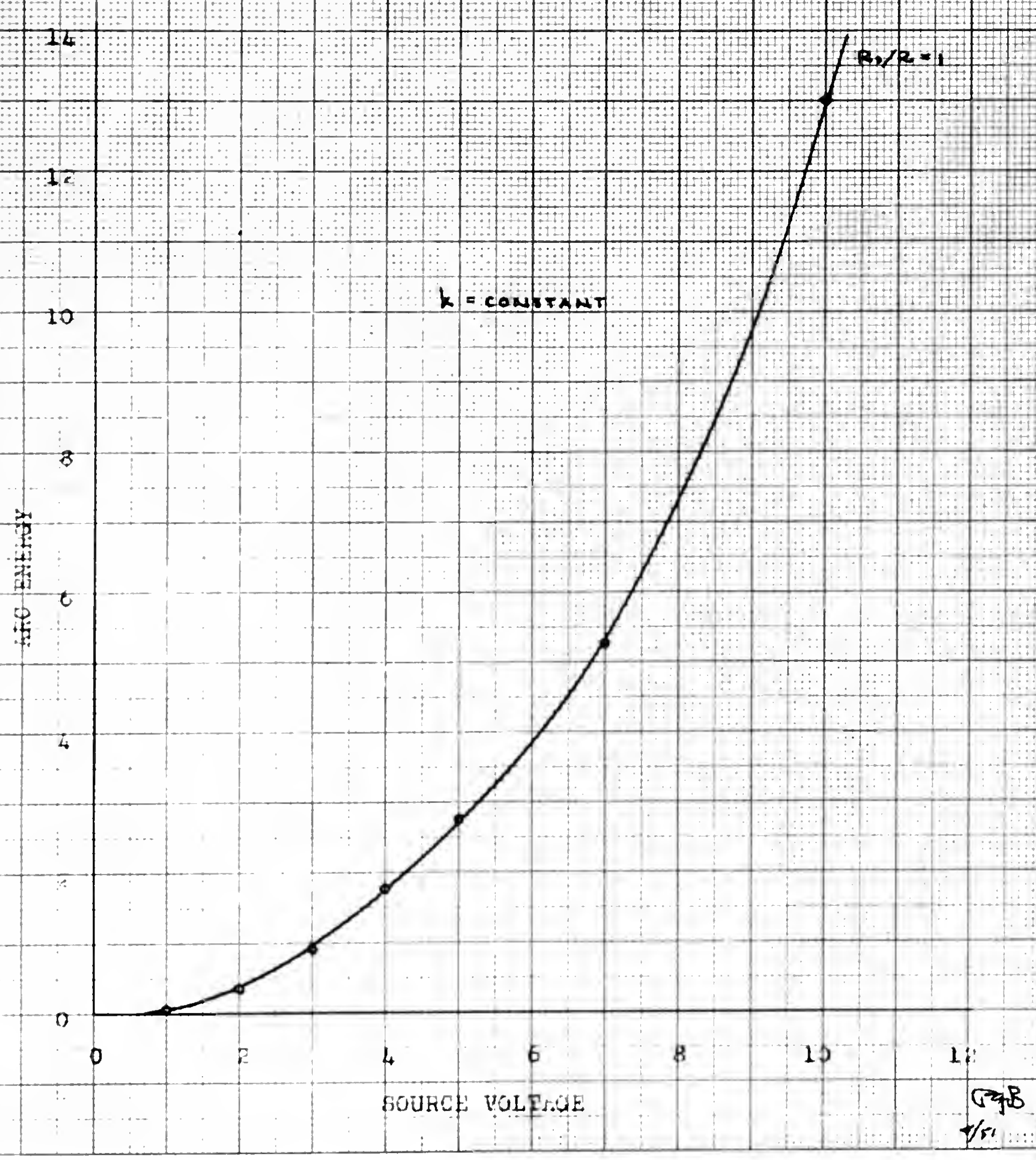


Figure XV shows the maximum voltage produced across the shunted fuse. In addition to its direct interest, this maximum voltage may be used, as shown in Appendix E, to compute the critical Fuse 2 melting energy for Case III.

Figure XXVI shows the variation of arc energy with circuit resistance for Case II. These calculations do not show a condition of maximum energy, as was found in Case I, but this may be due to the limited extent of the contours. Again, this figure is only an illustrative example and will not hold for all cases. As the shunt resistance increases the results for Case II approach the results for Case I. Figure XXVII shows the effect of variation of circuit inductance. Here there is clearly defined a "most severe" condition. From such a study, a fuse designer can specify a single test condition which would produce the greatest arc energy for any variation of circuit inductance in expected service. Figure XXVIII compares Case II with Case I, showing a definite advantage enjoyed by the limiter of Case II. Of course, the limiter of Case II does not interrupt the entire circuit current, but it does limit the current; and if there is a circuit breaker in the system the limiter can keep the current from exceeding the interrupting capacity of the breaker. If the heat dissipation capacity of the shunt resistor is sufficiently great this limiter arrangement will permit restricted operation of the system as soon as the fault is cleared, eliminating the necessity of replacing the fusable element before minimum service can be restored. This is of definite value in naval applications.

Figure XXIX shows the variation of arc energy with source voltage

Figure XXV shows the maximum voltage produced across the shunted fuse. In addition to its direct interest, this maximum voltage may be used, as shown in Appendix E, to compute the critical Fuse 2 melting energy for Case III.

Figure XXVI shows the variation of arc energy with circuit resistance for Case II. These calculations do not show a condition of maximum energy, as was found in Case I, but this may be due to the limited extent of the contours. Again, this figure is only an illustrative example and will not hold for all cases. As the shunt resistance increases the results for Case II approach the results for Case I. Figure XXVII shows the effect of variation of circuit inductance. Here there is clearly defined a most severe condition. From such a study, a fuse designer can specify a single test condition which would produce the greatest arc energy for any variation of circuit inductance in expected service. Figure XXVIII compares Case II with Case I, showing a definite advantage enjoyed by the limiter of Case II. Of course, the limiter of Case II does not interrupt the entire circuit current, but it does limit the current; and if there is a circuit breaker in the system the limiter can keep the current from exceeding the interrupting capacity of the breaker. If the heat dissipation capacity of the shunt resistor is sufficiently great this limiter arrangement will permit restricted operation of the system as soon as the fault is cleared, eliminating the necessity of replacing the fusable element before minimum service can be restored. This is of definite value in naval applications.

Figure XXIX shows the variation of arc energy with source voltage for Case II. The energy varies almost as the square of the source voltage.

Normal Operation, Case III and IV

Cases III and IV can normally be considered as combinations of Cases I and II, as explained in the procedure is exactly the same as set forth for Case II. To make calculations for the second fuse we must consider that the resistance in the circuit has been increased by the value of the shunt resistance.

Figure XXX is a sample design study for a limiter to restrict the fault current to 30% of the available current. This study shows that there is a value of shunt resistance to produce the minimum total arc energy. This occurs when the arc energies of the two fuses are equal, and this should result in fuses of comparable size.

To insure that the limiters of Cases III and IV operate normally the melting energy of Fuse II must be greater than the critical value. For Case III the critical value can be calculated, as shown in Appendix C, from the formula:

Critical M_2 (maximum $e_b)^3 / k_1 R_s^{\ 2}$; where maximum e_b is taken from Figure XV

Table I gives values of critical melting energy for Fuse 2 of Case IV.

Early Melting, Case III and IV

In Case III, if Fuse 2 should melt early the arc energy in Fuse 1 would be increased. However, here it is relatively easy to design to avoid early melting since Fuse 2 does not carry any current until Fuse 1 has melted.

for Case II. The energy varies almost as the square of the source voltage.

<u>Normal Operation, Case III and IV</u>

Cases III and IV can normally be considered as combinations of Cases I and II, as explained in the procedure is exactly the same as set forth for Case II. To make calculations for the second fuse we must consider that the resistance in the circuit has been increased by the value of the shunt resistance.

Figure X-X is a sample design study for a limiter to restrict the fault current to 30% of the available current. This study shows that there is a value of shunt resistance to produce the minimum total arc energy. This occurs when the arc energies of the two fuses are equal, and this should result in fuses of comparable size.

To insure that the limiters of Cases III and IV operate normally the melting energy of Fuse II must be greater than the critical value. For Case III the critical value can be calculated, as shown in Appendix I, from the formula:

Critical M_2 (maximum $e_2^2 \times R_s^2$, where maximum e_2 is taken from Fig. XV)

Table I gives values of critical melting energy for Fuse II of Case III.

<u>Early Melting, Case III and IV</u>

In Case III, if Fuse 2 should melt early the arc energy in Fuse 1 would be increased. However, here it is relatively easy to design to avoid early melting since Fuse 2 does not carry any current until Fuse 1 has melted.

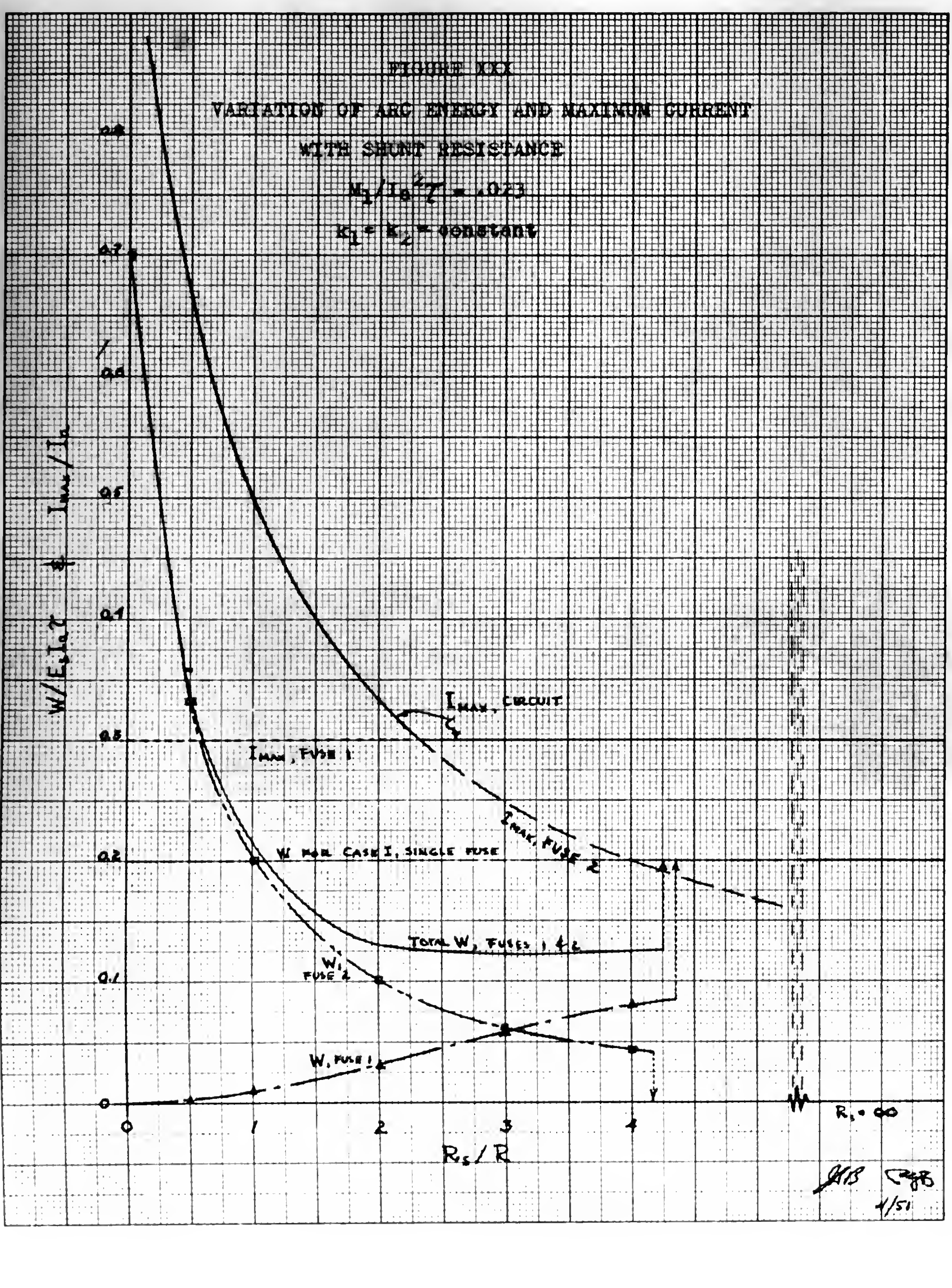


FIGURE XXI
VARIATION OF ARC ENERGY AND MAXIMUM CURRENT
WITH SHUNT RESISTANCE
$W_1/I_0^2\tau = .023$
$K_1 = K_2 =$ constant

In Case IV, early melting is more apt to occur since Fuse 2 must carry the total circuit current. Figures XVIII through XX show, for Case IV, the effect of early melting of Fuse 2 on Fuse 2 arc energy. Note that early melting generally increases the arc energy of Fuse 2, while it is apparent that the arc energy of Fuse 1 is reduced.

Since the early melting condition is undesirable for our purposes, the voluminous study necessary for a more complete analysis was not made. If a limiter is designed to avoid the early melting area under conditions of maximum fault current, it will avoid the area under all conditions. This is readily apparent from Figures XVIII through XX if we consider that the ratio of the melting energies of the two fuses is constant and that a decrease in available current always moves the point of operation to the right.

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CONCLUSIONS

We have shown that a map can be made showing variation of fuse arc energy, maximum voltage, and maximum current. We have constructed such a map for a particular form of fuse arc voltage. We have shown how it can be used to evaluate the difficulty of interruption, and to determine the effect of separate variables. The results are applicable to any fuse whose voltage waveform can be adequately approximated by the linearly rising voltage waveform assumed. We have shown that if the fuse voltage waveform can be identified as a function of circuit variables, the performance of the fuse can be predicted within the range of the contour calculations.

A shunting resistor lowers the requirement that must be met by a fuse. This combination is extremely advantageous when used in conjunction with a circuit breaker of interrupting capacity inadequate to handle the maximum short circuit current. In particular, we feel that this type of limiter may be used to advantage in storage battery submarine propulsion systems, where weight and space are at a premium and continuity of service is important.

A three element current limiting fuse retains the basic advantage of the shunting resistor. We have compared the two types of three element limiters and shown that the second fuse of a series three element limiter must have a large melting energy, and yet must melt after the current has been reduced to a fraction of the short circuit current. The parallel three

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RECOMMENDATIONS

We recommend that the results be used in the comparison of fuse interruptions to minimize the development testing required. We recommend that this or similar maps be combined with such functional relations for fuse voltage as can be obtained, to predict the most severe conditions a fuse will encounter.

We recommend that an attempt be made to exploit the advantages shown by our analysis for resistor-shunted limiter arrangements. In particular, an investigation should be made to determine the effect of a shunting resistor on the fuse arc voltage.

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APPENDIX

APPENDIX A

APPENDIX A - DETAILS OF PROCEDURE

Graphical Calculations

Sample calculations for Case I, Case II, and Case IV are contained in Appendix B.

The graphical calculations of arc energy and maximum fuse voltage for Case I proceeded as follows: i_f/I_a was plotted for a voltage step applied at fault time ($t/\tau = 0$). The back current, i_b, was plotted for a linearly rising voltage of slope $k\,\tau/E_s$, beginning at the time of melting. The difference between the forward and backward currents was plotted as net fuse current (i_n), the interruption being completed when i_n became zero. A curve of the product of fuse voltage (e_b) and net fuse current (i_n) was drawn and integrated by planimeter to evaluate the arc energy ($W/E_s I_a \tau$). Maximum fuse voltage was read directly for the time of net current zero.

For Case II, the method was identical, with the addition of the current component in the shunt resistor to the summation for net fuse current.

For the early melting Case IV calculations, the net Fuse 2 current was obtained by subtracting the back current (i_c) for Fuse 2 voltage (e_c) from the difference of i_f and i_b in Case II. The latter components ($i_f - i_b$) had to be adjusted for the removal of Fuse 1 voltage and the increase in circuit resistance at the instant of completion of Fuse 1 interruption. This interruption time occurred when the summation of i_f, i_b, i_r and i_c components through Fuse 1 reached zero. The reduction of Fuse 1 arc energy from the value for Case II was not computed.

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For Case II, the method was identical, with the addition of the current component in the shunt resistor to the summation for net fuse current.

For the early melting Case IV calculations, the net Fuse 2 current was obtained by subtracting the back current (i_2) for Fuse 2 voltage (e_2) from the difference of i_f and i_b in Case II. The latter components ($i_f - i_b$) had to be adjusted for the removal of Fuse 1 voltage and the increase in circuit resistance at the instant of completion of Fuse 1 interruption. This interruption time occurred when the summation of i_f, i_b, i_r and i_1 components through Fuse 1 reached zero. The reduction of Fuse 1 arc energy from the value for Case II was not computed.

Melting energies for Cases I and II were computed by integrating the curve squared forward current (i_f^2) from zero time to the time of melting. For Fuse 2 of Case IV, the squared difference of i_f and i_b was similarly integrated. For Fuse 2 of Case III, the melting energy is calculated from the current through the shunt resistor (i_r). Since this is a linearly rising current, proportional to Fuse 1 voltage, the melting energy is computed directly as a function of the shunt resistance, Fuse 1 voltage slope and maximum value (see Appendix C).

Graphical [illegible]

Appendix [illegible]

The [illegible]

for Case [illegible]

applied [illegible]

linearly [illegible]

The diff[illegible]

as seen [illegible]

[illegible]

was [illegible]

[illegible]

[illegible]

was [illegible]

from [illegible]

and [illegible]

[illegible]

This [illegible]

[illegible]

Fuse [illegible]

Basic Formulae for Graphical Calculation of Arc Energy

$I_a = E_s/R \qquad \tau = L/R \qquad n = K\tau/E_s$

$$i_f/I_a = \left(1 - \epsilon^{-t/\tau}\right)$$

$$e_b/E_s = n\left(\frac{t}{\tau} - \frac{T_M}{\tau}\right)$$

$$i_b/I_a = n\left(\frac{t}{\tau} - \frac{T_M}{\tau} + \epsilon^{-\left[\frac{t}{\tau} - \frac{T_M}{\tau}\right]} - 1\right)$$

Case I: $i_n = i_f - i_b$; $P = e_b i_n$; $W = \int e_b i_n \, dt$

$$i_n = I_a\left[\left(1-\epsilon^{-\frac{t}{\tau}}\right) - n\left(\frac{t}{\tau} - \frac{T_M}{\tau} + \epsilon^{-\left[\frac{t}{\tau} - \frac{T_M}{\tau}\right]} - 1\right)\right]$$

Case II: $i_n = i_f - i_b - i_r$

$$\frac{i_r}{I_a} = n\,\frac{R}{R_s}\left(\frac{t}{\tau} - \frac{T_M}{\tau}\right)$$

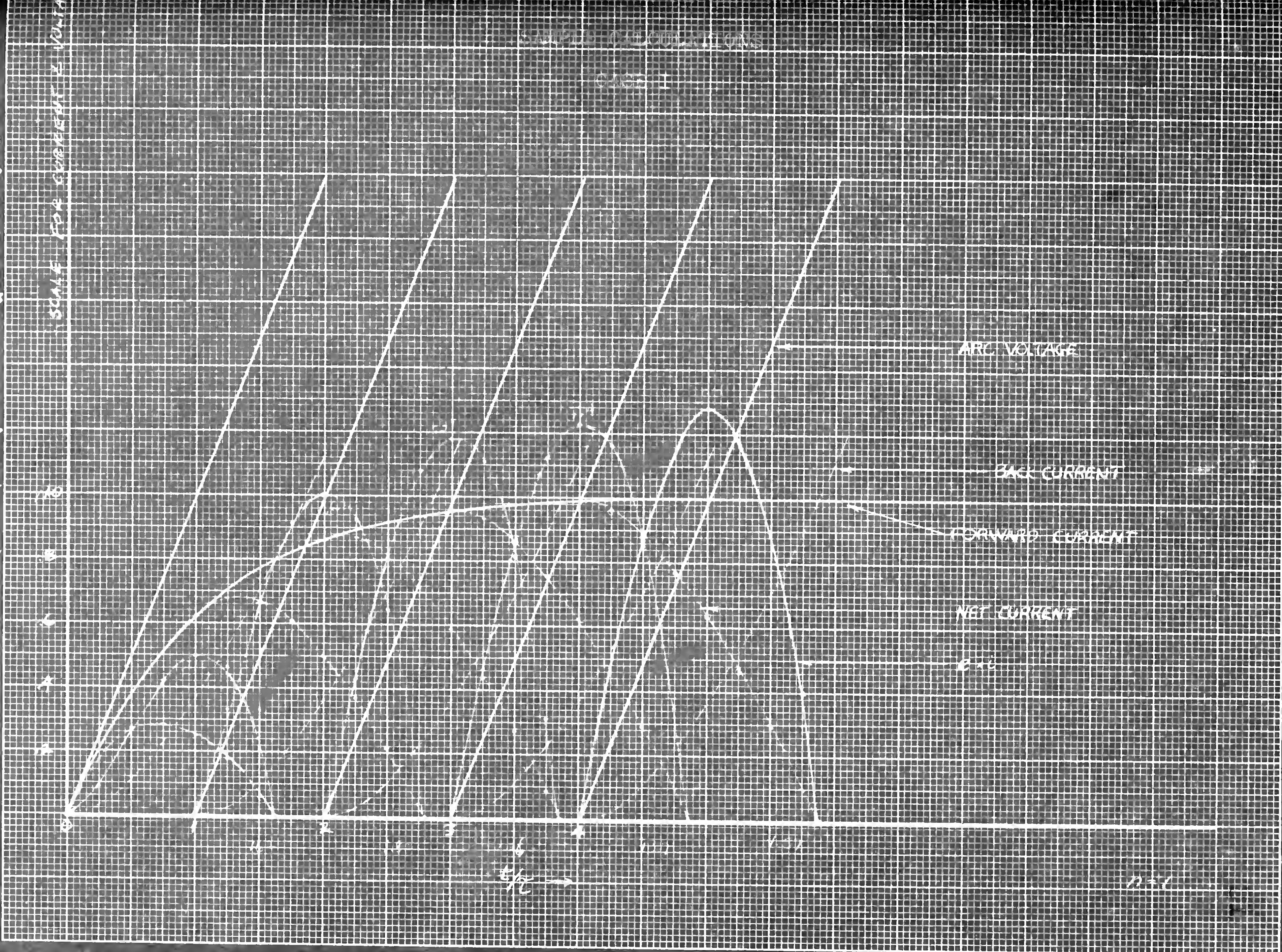
$$i_n = I_a\left[\left(1-\epsilon^{-\frac{t}{\tau}}\right) - n\left[\left(1+\frac{R}{R_s}\right)\left(\frac{t}{\tau} - \frac{T_M}{\tau}\right) + \epsilon^{-\left(\frac{t}{\tau} - \frac{T_M}{\tau}\right)} - 1\right]\right]$$

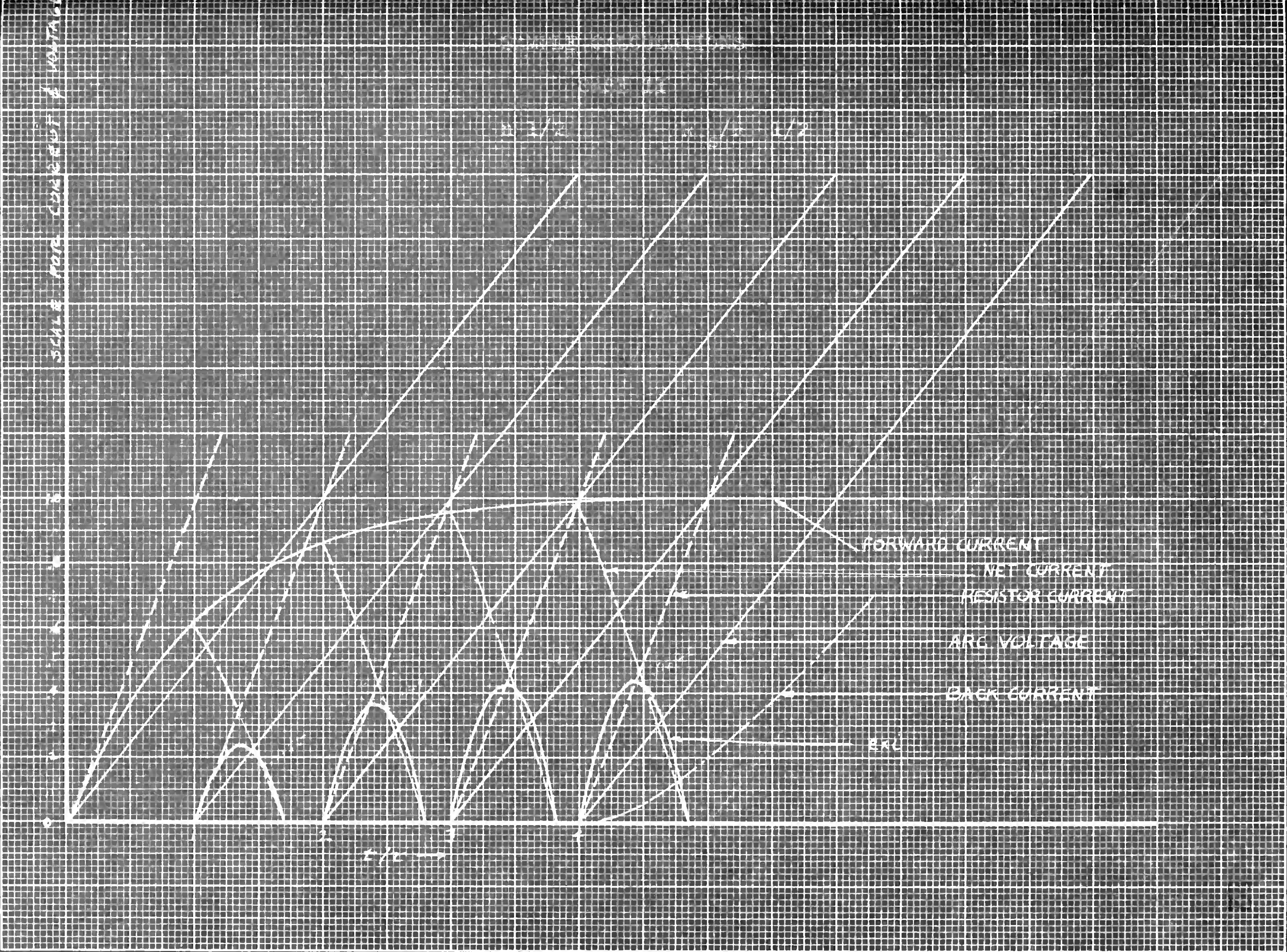
APPENDIX B - SAMPLE CALCULATIONS

Subject | Page

Sample graphical calculations of arc energy, maximum fuse voltage, and melting energies:

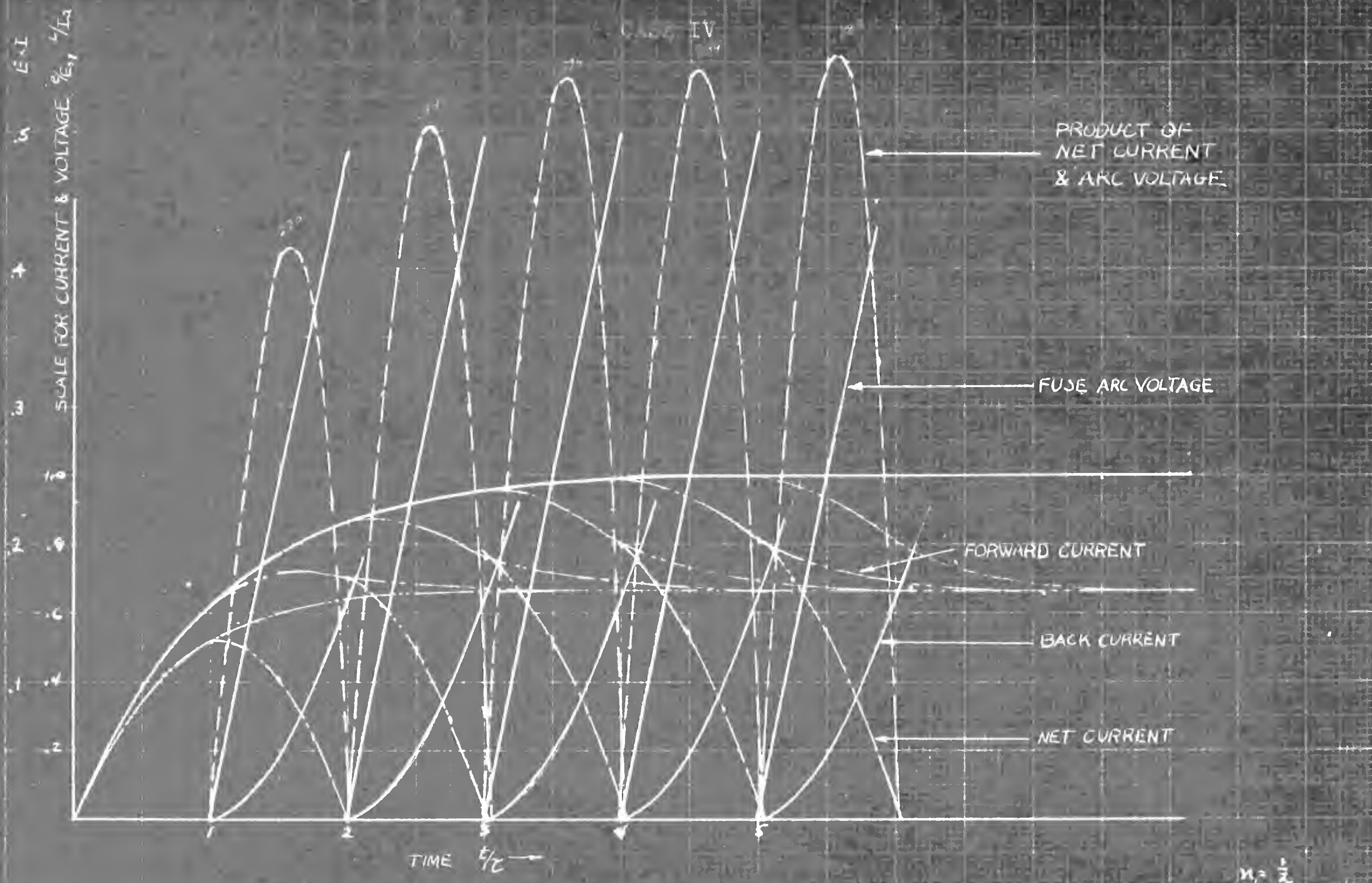
Calculations for derived curves:


ARC VOLTAGE
BACK CURRENT
FORWARD CURRENT
NET CURRENT
EI
n=1


SCALE FOR CURRENT & VOLTAGE
FORWARD CURRENT
NET CURRENT
RESISTOR CURRENT
ARC VOLTAGE
BACK CURRENT
EI
A
.3
.2
.1
0
1
2
3
4

SAMPLE CALCULATIONS

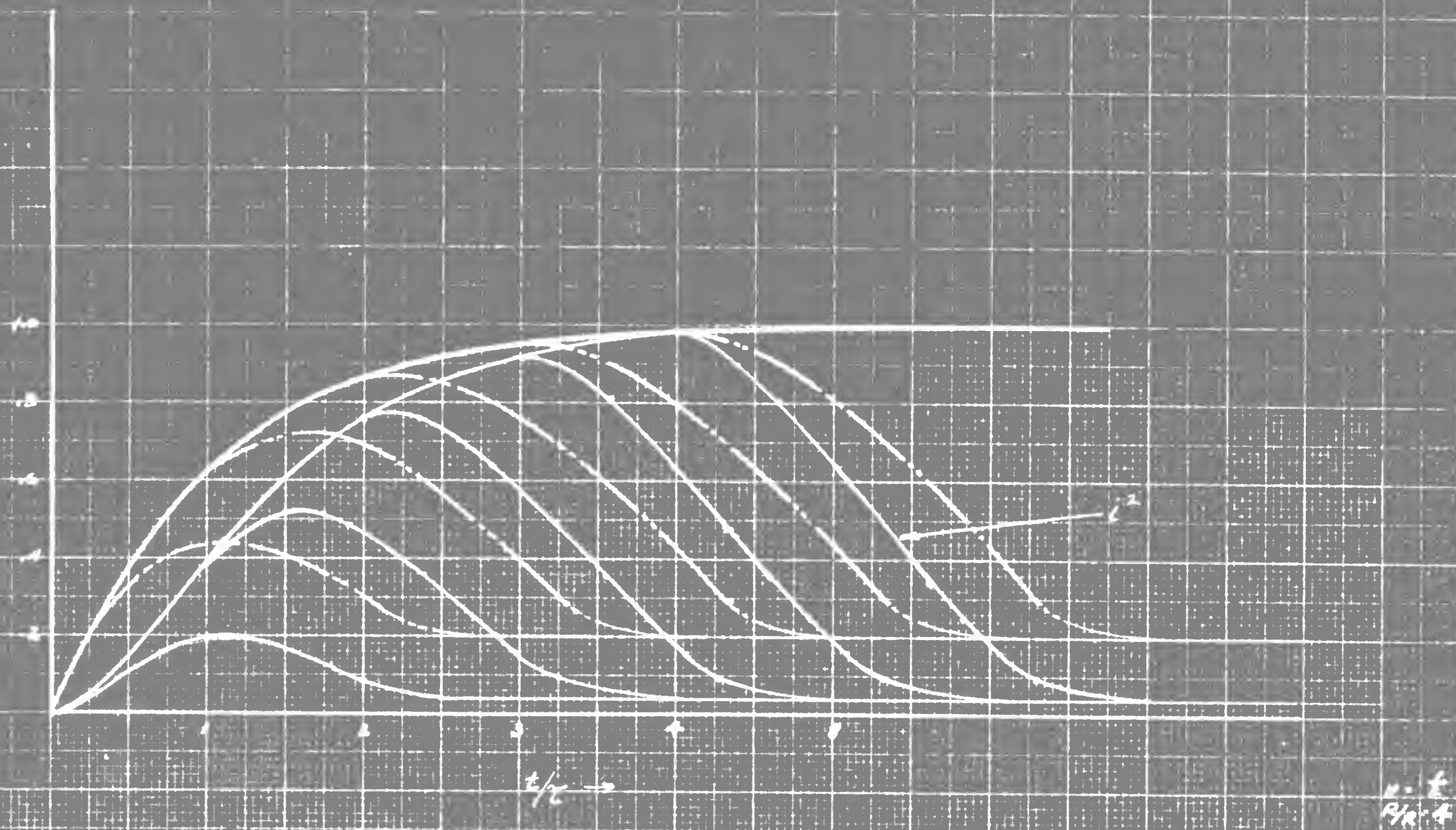
CASE IV



$n_1 = \frac{1}{2}$
$R/R = \frac{1}{2}$
$n_2 = 2$
$T_{H_2} = T_{H_1} + \tau$

SAMPLE CALCULATIONS

METHOD OF CALCULATING MELTING ENERGIES



CALCULATIONS FOR DERIVED CURVES
FIGURE XXIII

k_1 = constant , E_s = constant , L = constant

$\tau \propto \frac{1}{R}$, $I_a \propto \frac{1}{R}$ $\frac{m}{I_a^2 \tau} \propto R^3$

$m = \frac{k\tau}{E_s} \propto \frac{1}{R}$

R	R^3	m	$m/I_a^2\tau$	WR^2	R^2	W	E_{max}/E_s
1	1	2	.02	.20	1	.2	1.9
2	8	1	.16	.610	4	.152	1.85
3	27	.667	.54	.844	9	.0935	1.65
4	64	.5	1.28	.98	16	.081	1.495
5	125	.4	2.50	1.09	25	.0437	1.3

$k_1 \propto I_a$, $m \propto \frac{1}{R^2}$

R	R^2	R^3	m	$m/I_a^2\tau$	$W/E_s I_a \tau$	W	E_{max}/E_s
.91	.833	.75	6	.017			2.15
1	1	1	5	.0235	.1	.1	2.35
1.2	1.44	1.78	3.48	.0406	.196	.136	2.3
1.5	2.25	3.38	2.22	.0795	.35	.1555	2.2
2	4	8	1.2	.2	.59	.15	1.9
3	9	27	.555	.637	.93	.103	1.55
4	16	64	.315	1.505	1.195	.0744	1.38

CALCULATION FOR FIGURE XXIV

$E_s I_a \tau \propto E^2 \qquad I_a \propto E \qquad M/I_a^2\tau \propto \frac{1}{E^2}$

$k = \text{constant} \qquad n = \frac{k\tau}{E_s} \propto \frac{1}{E}$

E	E^2	n	$M/I_a^2\tau$	$\frac{W}{E_s I_a \tau}$	W	E_m/E_s	E_{max}
1	1	5	2.5	.585	.585		
2	4	2.5	.625	[illegible]	[illegible]	[illegible]	[illegible]
3	9	1.67	[illegible]	.276	[illegible]	[illegible]	[illegible]
4	16	1.25	.156	[illegible]	[illegible]	[illegible]	[illegible]
5	25	1	.100	[illegible]	[illegible]	[illegible]	[illegible]
7	49	.715	[illegible]	[illegible]	[illegible]	[illegible]	
10	100	.50	[illegible]	[illegible]	[illegible]	[illegible]	
11.38	126.5	.44	[illegible]		[illegible]	1.4	[illegible]

$k \propto I_a \qquad n \propto \frac{k\tau}{E_s} = \text{constant}$

E	E^2	n	$M/I_a^2\tau$	$\frac{W}{E_s I_a \tau}$	W	E/E_s	E_{max}
1	1	5	[illegible]	.585	.585	[illegible]	[illegible]
2	4	5	[illegible]	.45	1.8	[illegible]	[illegible]
3	9	5	.278	.38	[illegible]	3.1	[illegible]
4	16	5	.156	[illegible]	5.12	[illegible]	11.98
5	25	5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	49	5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	100	5	.025	[illegible]	[illegible]	[illegible]	[illegible]

CALCULATION FOR FIGURE XXV

λ = constant or function of I_0
Vary L, $\tau \propto L$ $m/I_0\tau \propto \frac{1}{L}$ E_s = constant
$n \propto L$

L	$M/I_0\tau$	n	$\frac{W}{E_s I_0 \tau}$	W	E_{max}/E_s
1	2	.31	1.20	1.2	1.35
2	1	.62	.88	1.76	1.61
3	.67	.93	.77	2.31	1.8
6	.33	1.86	.56	3.36	2.25
12	.167	3.72	.38	4.26	2.65
16	.125	4.96	.29	4.64	2.9

CALCULATION FOR FIGURE XXVIII

CASE II, VARY L, E_s = Const R/K = Constant

$n \propto L$ $m/I_0\tau \propto 1/L$

			$R_1/R = 1$		$R_1/R = 2$	
L	$M/I_0\tau$	n	$\frac{W}{E_s I_0 \tau}$	W	$\frac{W}{E_s I_0 \tau}$	W
.8	2.5	.248	.275	.22	.54	.432
1	2	.31	.248	.248	.475	.475
2	1	.62	.141	.282	.293	.586
3	.67	.93	.099	.297	.203	.609
4	.5	1.24	.066	.264	.152	.608
5	.4	1.55	.049	.245	.123	.615
6	.33	1.86	.0365	.219	.103	.618
8	.25	2.48	.024	.192	.072	.576
10	.2	3.10	.017	.17	.0575	.575
12	.167	4.96	.012	.144		

CALCULATIONS FOR FIGURE XXVI

VARY R, k = CONSTANT }: E_s = CONSTANT, L = CONSTANT; $\tau \sim \frac{1}{R}$, $I_a \sim \frac{1}{R}$; $M/I_a^2\tau \sim R^3$, $k\tau/E_s \sim \frac{1}{R}$

R	$1/R$	R^2	R^3	R_s/R	$k\tau/E_s$	$M/I_a^2\tau$	$W/k_sI_a\tau$	W
1	1	1	1	3.72	2	.02	.07	.07
1.33	.75	1.7[illegible]	2.36	[illegible]	1.5	.0472	.082	.048[illegible]
2	[illegible]	4	8	1.86	1.0	.16	.122	[illegible]
3	.33	9	27	1.24	0.66	.54	.148	.0165
4	.25	16	64	.93	0.50	1.28	.147	.0092

VARY R, $K \sim I_a$ }: E_s = CONSTANT, L = CONSTANT; $\tau \sim \frac{1}{R}$, $I_a \sim \frac{1}{R}$; $M/I_a^2\tau \sim R^3$, $K\tau/E_s \sim \frac{1}{R^2}$

R	$1/R$	R^2	R^3	R_s/R	$K\tau/E_s$	$M/I_a^2\tau$	$W/E_sI_a\tau$	W
1	1	1	1	3.72	2	.02	.07	.07
1.414	0.71	2	2.83	2.64	1	.0567	.120	.0695
2	0.5	4	8	1.86	0.5	.16	[illegible]	.0567
2.83	0.354	8	[illegible]	1.31	.125	.42	.32[illegible]	.0407

CALCULATIONS FOR FIGURE XXIX

VARY E_1 }; K = CONSTANT }

$I_a \sim E_1$ $\qquad$ $E_1 I_a \tau \sim E_1^2$

$n = K\tau/E_1 \sim \frac{1}{E_1}$ $\qquad$ $M/I_a^2\tau \sim 1/E_1^2$

E_1	E_1^2	n	$M/I_a^2\tau$	$W/E I_a \tau$	W
1	1	2	2.5	.059	.059
2	4	1	.625	.0925	.370
3	9	.67	.278	.101	.909
4	16	.5	.156	.111	1.776
5	25	.4	.100	.110	2.75
7	49	.286	.051	.135	5.27
10	100	.2	.025	.130	13.0
11.35	128.5	.176	.0194		

CALCULATIONS FOR FIGURE XXX

$K_1 = K_2$

$n_1 = K_1\tau/E_1 = 2$

$M_1/I_a^2\tau = .025$

$(E_1 I_a \tau)_2 \sim \left(\frac{1}{R_1+R_2}\right)^2 = \left(\frac{1}{R_2}\right)^2$

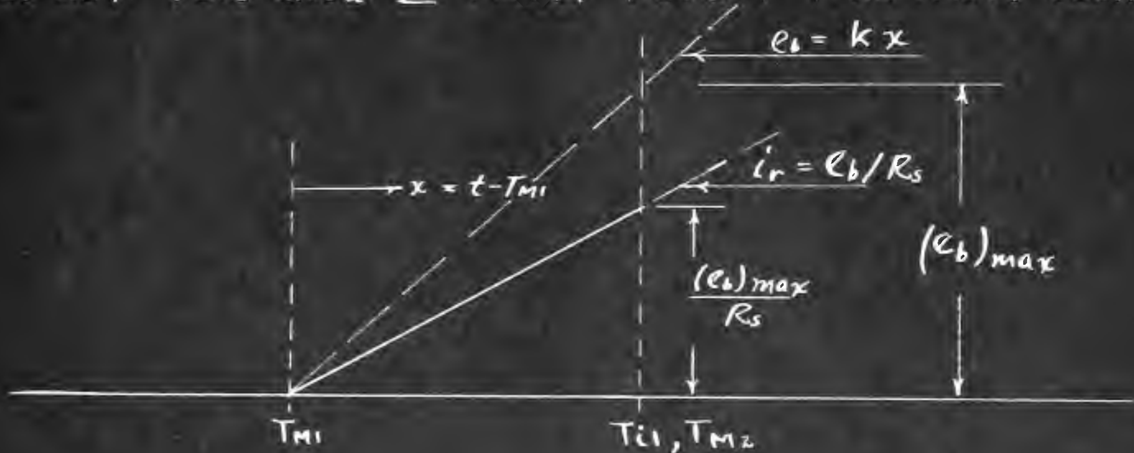
$n_2 = K_2\tau/E_1 \sim \frac{1}{R_1+R_2} = \frac{1}{R_2}$

R_3/R_1	R_2	$\frac{W_1}{E_1 I_a \tau}$	$\frac{W_2}{(E_1 I_a \tau)_2}$	$\frac{W_2}{E_1 I_a \tau}$	$\frac{W_1+W_2}{E_1 I_a \tau}$
1/2	1½	.0016	.75	.555	.556
1	2	.01	.80	.20	.21
2	3	.03	.90	.10	.13
3	4	.06	1.00	[illegible]	[illegible]
4	5	.08	1.10	.044	.124
∞	— CASE I —				.50

$(L/I_a)_{min} = 0.50$

APPENDIX C - SUPPLEMENT TO DISCUSSION

COMPUTATION OF CRITICAL FUSE 2 MELTING ENERGY FOR CASE III FROM FUSE 1 MAXIMUM VOLTAGE.



FUSE 1 MAXIMUM VOLTAGE $[(e_b)_{max}]$ IS TAKEN FROM FIGURE XV (CASE II).

FOR CRITICAL MELTING ENERGY, $T_{M2} = T_{i1}$

$$\text{CRITICAL } M_2 = \int_{T_M}^{T_i} i_r^2\,dt = \int_0^{T_M - T_i} i_r^2\,dx$$

$$i_r = kx/R_s \quad ; \quad dx = R_s\,di_r/k$$

$$\text{CRITICAL } M_2 = \frac{R_s}{k}\int_{i(T_M)}^{i(T_i)} i^2\,di = \frac{R_s}{3k}\, i^3 \Big|_{i(T_M)}^{i(T_i)}$$

$$i(T_i) = \frac{1}{R_s}(e_b)_{max} \; ; \quad i(T_M) = 0$$

$$\boxed{\text{CRITICAL } M_2 = (e_b)^3_{max} / 3kR_s^2}$$

APPENDIX D - ORIGINAL DATA

CASE I

n	$W/E_sI_a\tau$, t/τ = 0	1	2	3	4	E_{max}/E_s, t/τ = 0	1	2	3	4	n
1/4	1.05	1.21	1.25	1.29	1.30	1.25	1.25	1.25	1.25	1.25	1/4
1/2	.566	.841	.934	.976	.977	1.41	1.48	1.48	1.48	1.48	1/2
1	.248	.600	.739	.774	.801	1.62	1.80	1.86	1.87	1.87	1
2	.095	.448	.608	.680	.700	1.82	2.30	2.38	2.46	2.48	2
5	.013	.302	.466	.536	.580	1.75	3.00	3.35	3.50	3.50	5

CASE II

n	$R_s/R = 1/2$, t/τ = 0	1	2	3	4		$R_s/R = 1$, t/τ = 0	1	2	3	4	n
1/4	.0032	.051	.0745	.0824	.0824	$\frac{W}{E_sI_a\tau}$	.0745	.216	.264	.276	.276	1/4
1/2	0	.0255	.049	.0587	.0626		.0068	.108	.146	.168	.176	1/2
1	0	.0127	.0255	.0294	.0324		0	.055	.100	.117	.118	1
2	0	.0059	.0117	.0196	.0216		0	.0235	.0431	.0569	.0569	2
1/4	.19	.32	.34	.34	.34	$\frac{E_{max}}{E_s}$	.56	.60	.61	.61	.61	1/4
1/2	0	.35	.40	.415	.425		.43	.63	.68	.69	.69	1/2
1	0	.34	.42	.44	.45		0	.65	.75	.78	.78	1
2	0	.32	.42	.48	.48		0	.62	.78	.82	.82	2

n	$R_s/R = 2$, t/τ = 0	1	2	3	4		$R_s/R = 4$, t/τ = 0	1	2	3	4	n
1/4	.308	.446	.495	.514	.514	$\frac{W}{E_sI_a\tau}$	.447	.602	.651	.669	.669	1/4
1/2	.0825	.253	.306	.339	.346		.204	.424	.508	.557	.546	1/2
1	.0043	.131	.198	.222	.259		.0431	.259	.355	.382	.404	1
2	0	.0706	.126	.149	.166		.001	.1585	.249	.288	.294	2
1/4	.80	.82	.82	.83	.83	$\frac{E_{max}}{E_s}$	.93	.93	.93	.93	.93	1/4
1/2	.77	.90	.95	.95	.95		1.05	1.15	1.17	1.17	1.17	1/2
1	.60	1.00	1.10	1.10	1.12		1.00	1.31	1.40	1.42	1.42	1
2	0	1.10	1.25	1.32	1.32		.55	1.52	1.68	1.70	1.70	2

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